केन्द्रीय विद्यालय संगठन, क्षेत्रीय कार्यालय एर्नाकुलम

KENDRIYA VIDYALAYA SANGATHAN
ERNAKULAM REGION

MINIMUM LEVEL LEARNING MATERIAL
SESSION- 2022-23

तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

CLASS X
MATHEMATICS

KENDRIYA VIDYALAYA SANGATHAN ERNAKULAM REGION

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CONTENT INDEX

CLASS X -MATHEMATICS

MATHEMATICS (CODE NO. 041)

COURSE STRUCTURE CLASS –X

UNITS	UNIT NAME	MARKS
I	NUMBER SYSTEM	6
II	ALGEBRA	20
III	CO-ORDINATE GEOMETRY	6
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILITY	11
	TOTAL	80
	INTERNAL ASSESSMENT	20
	TOTAL	100

INTERNAL ASSESSMENT

INTERNAL ASSESSMENT	marks	TOTAL MARKS
Pen Paper Test and Multiple Assessment (5+5)	10marks	20 marks
Portfolio	05 marks	
Lab Practical (Lab activities to be done from the prescribed books)	05 marks	

REAL NUMBERS

Important Concepts:

1. The Fundamental Theorem of Arithmetic

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur. This fact is also stated as,

The prime factorisation of a natural number is unique, except for the order of its factors.

2. Property of HCF and LCM of two positive integers 'a' and 'b':

HCF (a, b) × LCM (a, b) = a ×b

LCM (a, b) = $\frac{a \times b}{HCF(a,b)}$

HCF (a, b) = $\frac{a \times b}{LCM(a,b)}$

3. Prime factorisation method to find HCF and LCM

HCF (a, b) = Product of the smallest power of each common prime factor in the numbers.
LCM (a, b) = Product of the greatest power of each prime factor, involved in the numbers.

4. Proofs of irrationality of √2, √3, √5 etc.

IMPORTANT QUESTIONS

PART – A (Each question carries 1 Mark)

1. Let a and b be two positive integers such that a = $p^3q^4$and b = p^2q^3, where p and q are prime numbers. If HCF(a,b) = $p^m\, q^n$ and LCM(a,b) = $p^r\, q^s$, then (m+n)(r+s)=

   (a) 15 (b) 30 (c) 35 (d) 72

2. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers.

3. What is the greatest possible speed at which a man can walk 52 km and 91 km in an exact number of hours?

   (a) 17 km/hours (b) 7 km/hours (c) 13 km/hours (d) 26 km/hours

4. If HCF and LCM of two numbers are 4 and 9696, then the product of the two numbers is

   (a) 9696 (b) 24242 (c) 38784 (d) 4848

5. If a and b are positive integers; then HCF (a,b) x LCM (a,b)

   (a) a x b (b) a + b (c) a - b (d) a/b

PART – B (Each question carries 2 Marks)

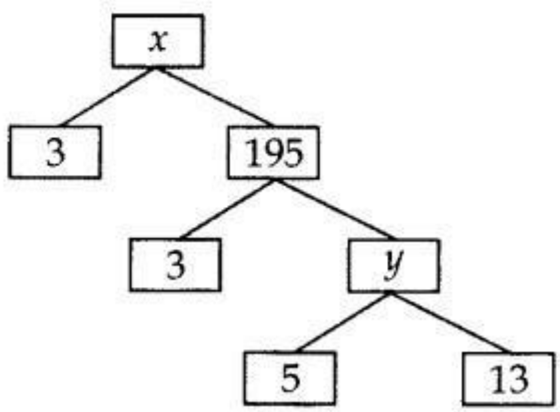
6. HCF and LCM of two numbers are 9 and 459 respectively. If one of the numbers is 27, find the other number.

7. Find HCF and LCM of 13 and 17 by prime factorisation method.

8. Find LCM of numbers whose prime factorisation are expressible as 3×5^2 and $3^2 \times 7^2$.

9. Find the LCM of 96 and 360 by using the fundamental theorem of arithmetic.

10. Complete the following factor tree and find the composite number



PART – C (Each question carries 3 Marks)

11. Prove that $2 + 3\sqrt{5}$ is an irrational number.
12. Show that $3\sqrt{7}$ is an irrational number.
13. Prove that $\sqrt{5}$ is irrational and hence show that $3 + \sqrt{5}$ is also irrational.

14. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?
15. An army contingent of 1000 members is to march behind an army band of 56 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

PART – D (Each question carries 5 Marks)

16. The length, breadth, and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.
17. In a school, there are two Sections A and B of class X. There are 48 students in Section A and 60 students in Section B. Determine the least number of books required for the library of the school so that the books can be distributed equally among all students of each Section.
18. Dudhnath has two vessels containing 720 ml and 405 ml of milk respectively. Milk from these containers is poured into glasses of equal capacity to their brim. Find the minimum number of glasses that can be filled.

19. Find the HCF and LCM of 306 and 657 and verify that LCM × HCF = Product of the two numbers.
20. Two tankers contain 850 liters and 680 liters of petrol. Find the maximum capacity of a container which can measure the petrol of each tanker in the exact number of times

ANSWERS

Part – A

Q. 1) It is given in the question that,

a and b be two positive integers such that **a =** p^3q^4 and **b=** p^2q^3 , where p and q are prime numbers.

Now,

HCF i.e., Highest common factor of (a, b) will be, p^2q^3 ....(1)

LCM i.e., Lowest common factor of (a, b) will be, p^3q^4 . ..._(2)

It is given in the question that,

HCF$(a, b) = p^m q^n$ ....(3)and LCM$(a, b) = p^r q^s$...........(4)

Now comparing the given values of HCF and LCM with the determining values we get the values of the variables, or, comparing the equation (1) with equation (3) and equation (2) with equation (4) we get the values of the variables as follow,

m=2

n=3

r=3

s=4

Hence,

the ***value of (m + n) (r + s)*** will be,

$$(m+n)(r+s) = (2+3)(3+4)$$
$$(m+n)(r+s) = 5 \times 7$$
$$(m+n)(r+s) = 35$$

Hence, the ***correct answer*** is option (c) 35.

2. Prime factorisation of 404 = 2 × 2 × 101

Prime factorisation of 96 = 2 × 2 × 2 × 2 × 2 × 3 = 25 × 3

HCF = 2 × 2 = 4

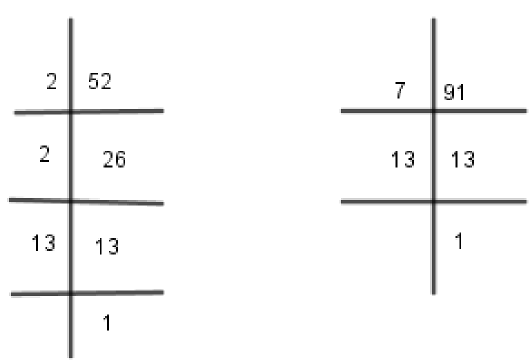
LCM = 25 × 3 × 101 = 9696

HCF × LCM = 4 × 9696 = 38784

Product of the given two numbers = 404 × 96 = 38784

Hence, verified that LCM × HCF = Product of the given two numbers.

3. Greatest possible speed = HCF of (52,91) ⇒



So,

HCF = 13 km/hr is the greatest possible speed.

4. HCF = 4

   LCM = 9696

   We know that,

   Product of two number = HCF × LCM

   Product of two number = 4 × 9696

   Product of two number = 38784

   Therefore, the product of two numbers is 38784.

5. There is an identity which holds for all integers:

   LCM(a,b)×HCF(a,b)=ab
   Hence, the correct option is (c)

PART B (2 marks each)

6. We know,
   1st number × 2nd number = HCF × LCM
   ⇒ 27 × 2nd number = 9 × 459
   ⇒ 2nd number = $\frac{9\times459}{27}$ = 153

7. 13 = 1 × 13; 17 = 1 × 17
   HCF = 1 and LCM = 13 × 17 = 221

8. LCM $(3 \times 5^2, 3^2 \times 7^2) = 3^2 \times 5^2 \times 7^2 = 9 \times 25 \times 49 = 11025$

9. $96 = 2^5 \times 3$
   $360 = 2^3 \times 3^2 \times 5$
   LCM $= 2^5 \times 3^2 \times 5 = 32 \times 9 \times 5 = 1440$

2	96
2	48
2	24
2	12
2	6
3	3

2	360
2	180
2	90
3	45
3	15
5	5

10. y = 5 × 13 = 65 x = 3 × 195 = 585

PART –C (3 marks each)

11. Let us assume, to the contrary, that $2 + 3\sqrt{5}$ is rational.

So that we can find integers a and b ($b \neq 0$).

Such that $2 + 3\sqrt{5}$ = ab, where a and b are coprime.

Rearranging the above equation, we get

$$3\sqrt{5} = \frac{a}{b} - 2$$

$$3\sqrt{5} = \frac{a - 2b}{b}$$

$$\sqrt{5} = \frac{a - 2b}{3b} = \frac{a}{3b} - \frac{2b}{3b}$$

$$\sqrt{5} = \frac{a}{3b} - \frac{2}{3}$$

Since a and b are integers, we get $\frac{a}{3b} - \frac{2}{3}$ is rational and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

So, we conclude that $2 + 3\sqrt{5}$ is irrational.

12. Let us assume, to the contrary, that $3\sqrt{7}$ is rational.

That is, we can find coprime a and b ($b \neq 0$) such that $3\sqrt{7} = \frac{a}{b}$

Rearranging, we get $\sqrt{7} = \frac{a}{3b}$

Since 3, a and b are integers, $\frac{a}{3b}$ is rational, and so $\sqrt{7}$ is rational.

But this contradicts the fact that $\sqrt{7}$ is irrational. So, we conclude that $3\sqrt{7}$ is irrational.

13. Let us assume, to the contrary, that $\sqrt{5}$ is rational.

So, we can find integers p and q ($q \neq 0$),

such that $\sqrt{5} = \frac{p}{q}$, where p and q are coprime.

Squaring both sides, we get

$5 = \frac{p2}{q2} \Rightarrow 5q^2 = p^2$ …(i)

$\Rightarrow$ 5 divides p^2 5 divides p

So, let p = 5r

Putting the value of p in (i), we get

$\Rightarrow$

$5q^2 = (5r)^2 \Rightarrow 5q^2 =$

$25r^2$ $q^2 = 5r^2 \Rightarrow 5$

divides q^2

5 divides q

So, p and q have at least 5 as a common factor.

But this contradicts the fact that p and q have no common factor.

So, our assumption is wrong, √5 is irrational.

√5 is irrational, 3 is a rational number.

So, we conclude that 3 + √5 is irrational.

14. $9 = 3^2$, $12 = 2^2 \times 3$, $15 = 3 \times 5$

LCM = $2^2 \times 3^2 \times 5 = 4 \times 9 \times 5 = 180$ minutes or 3 hours.

They will next toll together after 3 hours.

15. 1000 =2x2x2x5x5x5

56 = 2x2x2x7

HCF of 1000 and 56 = 8

Maximum number of columns = 8.

PART – D (5 marks each)

16. To find the length of the longest rod that can measure the dimensions of the room exactly, we have to find HCF.

L, Length = 8 m 50 cm = 850 cm = $2^1 \times 5^2 \times 17$

B, Breadth = 6 m 25 cm = 625 cm = 5^4

H, Height = 4 m 75 cm = 475 cm = $5^2 \times 19$

HCF of L, B and H is 5^2 = 25 cm

Length of the longest rod = 25 cm

17. Since the books are to be distributed equally among the students of Section A and Section B. Therefore, the number of books must be a multiple of 48 as well as 60. Hence, the required number of books is the LCM of 48 and 60.

$48 = 2^4 \times 3$

$60 = 2^2 \times 3 \times 5$

LCM = $2^4 \times 3 \times 5 = 16 \times 15 = 240$

Hence, the required number of books is 240.

2	48		2	60
2	24		2	30
2	12		3	15
2	6			5
	3			

18. 1st vessel = 720 ml; 2nd vessel = 405 ml

We find the HCF of 720 and 405 to find the maximum quantity of milk to be filled in one glass. $405 = 3^4 \times 5$

$720 = 2^4 \times 3^2 \times 5$

HCF = $3^2 \times 5$ = 45 ml = Capacity of glass

No. of glasses filled from 1st vessel = 72045 = 16

No. of glasses filled from 2nd vessel = 40545 = 9

Total number of glasses = 25

19. $306 = 2 \times 3^2 \times 17$

$657 = 32 \times 73$

HCF = $3^2 = 9$

LCM = $2 \times 3^2 \times 17 \times 73 = 22338$

L.H.S. = LCM × HCF = 22338 × 9 = 201042

R.H.S. = Product of two numbers = 306 × 657 = 201042

L.H.S. = R.H.S.

20. To find the maximum capacity of a container which can measure the petrol of each tanker in the exact number of times, we find the HCF of 850 and 680.

$850 = 2 \times 5^2 \times 17$

$680 = 2^3 \times 5 \times 17$

HCF = $2 \times 5 \times 17 = 170$

Maximum capacity of the container = 170 litres.

2	850		2	680
5	425		2	340
5	85		2	170
	17		5	85
				17

POLYNOMIALS

SHORT NOTES

- "Polynomial" comes from the word 'Poly' (Meaning Many) and 'nomial' (in this case meaning Term)-so it means many terms.
- A polynomial is made up of terms that are only added, subtracted or multiplied.
- A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
- Degree – The highest exponent of the variable in the polynomial is called the degree of polynomial. Example: $3x^3 + 4$, here degree = 3.
- Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomial respectively.
- A polynomial can have terms which have Constants like 3, -20, etc., Variables like x and y and Exponents like 2 in y^2.
- The zeroes of a polynomial p(x) are precisely the x-coordinates of the points, where the graph of y = p(x) intersects the x-axis.

- If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then
  Sum of zeros,$\alpha+\beta=\frac{-b}{a}=\frac{-\, coefficient\ of\ x}{coefficient of x^2}$
  product of zeros,$\alpha\beta=\frac{c}{a}=\frac{constant\ term}{coefficient of x^2}$

- If r (x) = 0, then g (x) is a factor of p (x).

- Common Identity used in this chapter

$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$
$\therefore(\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$

SECTION A

1. If the sum of the zeroes of the polynomial $p(x) = (k^2 – 14) x^2 – 2x – 12$ is 1, then find the value of k.
2. If α and β are the zeroes of a polynomial such that α + β = -6 and αβ = 5, then find the polynomial.
3. If α and β are the zeroes of the polynomial $ax^2 + bx + c$, find the value of $\alpha^2 + \beta^2$.
4. Form a quadratic polynomial whose zeroes are $3 + \sqrt{2}$ and $3 - \sqrt{2}$
5. Find the condition that zeroes of polynomial $p(x) = ax^2 + bx + c$ are reciprocal of each other.

SECTION B

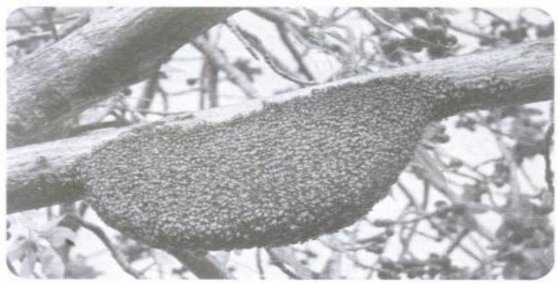
6. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 – 5x – 3$, find the value of p and q
7. Find the zeroes of $p(x) = 2x^2 – x – 6$ and verify the relationship of zeroes with these coefficients.
8. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 – x – 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$
9. If the square of the difference of the zeros of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of *p*.
10. Find the value of 'k' such that the quadratic polynomial $x^2 – (k + 6) x + 2 (2k +1)$ has sum of the zeros is half of their product.

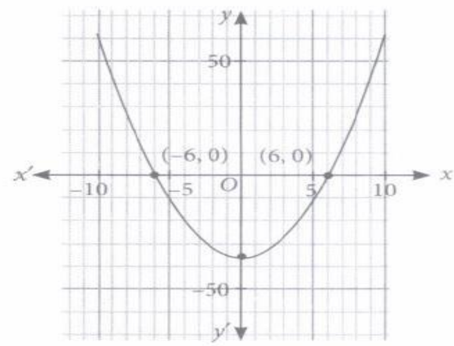
SECTION C

11. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 – p (x + 1) – c$, show that $(\alpha +1) (\beta +1) = 1 – c$.
12. Find the value of 'k' so that the zeros of the quadratic polynomial $3x^2 – kx + 14$ are in the ratio 7:6
13. Find the value of '*k*' for which the polynomial $x^4 + 10x^3 + 25x^2 + 15x + k$ is exactly divisible by (x + 7).
14. If αand β are the zeros of the polynomial $f (x) = x^2 + px + q$, find polynomial whose zeros are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$.
15. Find the zeros of the polynomial $p(x) = 4\sqrt{3}x^2 + 5x – 2\sqrt{3}$ and verify the relationship between the zeros and its coefficients.

SECTION D

16. While playing in the garden, Sahiba saw a honeycomb and asked her mother what is that. She replied that it's a honeycomb made by honey bees to store honey. Also, she told her that the shape of the honeycomb formed is parabolic. The mathematical representation of the honeycomb structure is shown in the graph.





Based on the above information, answer the following questions.
(i) Graph of a quadratic polynomial is in___________shape.
(a) straight line (b) parabolic
(c) circular (d) None of these
(ii) The expression of the polynomial represented by the graph is

(a) x^2-49 (b) x^2-64 (c) x^2-36 (d) x^2-81
(iii) Find the value of the polynomial represented by the graph when x = 6.

(a) -2 (b) -1 (c) 0 (d) 1
(iv) The sum of zeroes of the polynomial $x^2 + 2x - 3$ is
(a) -1 (b) -2 (c) 2 (d) 1
(v) If the sum of zeroes of polynomial $at^2 + 5t + 3a$ is equal to their product, then find the value of a.
(a) -5 (b) -3 (c) $\frac{5}{3}$ (d) $\frac{-5}{3}$

17. Pankaj's father gave him some money to buy avocado from the market at the rate of p(x) = $x^2 - 24x + 128$. Let , α and β are the zeroes of p(x).
Based on the above information, answer the following questions.

(i) Find the value of a and β, where a < β.

(a) -8, -16 (b) 8,16 (c) 8,15 (d) 4,9

(ii) Find the value of $\alpha + \beta + \alpha\beta$.

(a) 151 (b) 158 (c) 152 (d) 155

(iii) The value of p(2) is

(a) 80 (b) 81 (c) 83 (d) 84

(iv) If α and β are zeroes of $x^2 + x - 2$, then 1α+1β=x2+x−2, then $\frac{1}{\alpha} + \frac{1}{\beta}$

(a) ½ (b) 1/3 (c) 1/4 (d) 1/5

(v) If sum of zeroes of $q(x)= kx^2 + 2x +3k$ is equal to their product, then k =

(a) 2/3 (b) 1/3 (c) -2/3 (d) -1/3

18. If $p(x) = x^3 - 2x^2 + kx + 5$ is divided by $(x - 2)$, the remainder is 11. Find k. Hence find all the zeroes of $x^3 + kx^2 + 3x + 1$.
19. If α and β are zeroes of $p(x) = kx^2 + 4x + 4$, such that $\alpha^2 + \beta^2 = 24$, find k.
20. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$, satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = 214$ then find the value of k.

ANSWERS

SECTION A

1. $p(x) = (k^2 - 14)\,x^2 - 2x - 12$
   Here $a = k^2 - 14$, $b = -2$, $c = -12$
   Sum of the zeroes, $(\alpha + \beta) = 1$ …[Given]
   $\Rightarrow \frac{-b}{a} = 1$
   $\Rightarrow \frac{-(-2)}{(k2 - 14)} = 1$
   $\Rightarrow k^2 - 14 = 2$
   $\Rightarrow k^2 = 16$
   $\Rightarrow k = \pm 4$
2. Quadratic polynomial is $x^2 - (\alpha + \beta)\,x + \alpha\,\beta = 0$
   $\Rightarrow x^2 - (-6)x + 5 = 0$
   $\Rightarrow x^2 + 6x + 5 = 0$
3.

$\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\alpha^2 + \beta^2 = \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$$

$$\alpha^2 + \beta^2 = \frac{b^2}{a^2} - \frac{2c}{a}$$

$$\therefore \quad \alpha^2 + \beta^2 = \frac{b^2 - 2ca}{a^2}$$

4.

Sum of zeroes,

$S = (3 + \sqrt{2}) + (3 - \sqrt{2}) = 6$

Product of zeroes,

$P = (3 + \sqrt{2}) \times (3 - \sqrt{2}) = (3)^2 - (\sqrt{2})^2 = 9 - 2 = 7$

Quadratic polynomial = $x^2 - (\alpha + \beta)\, x + \alpha\, \beta = x^2 - 6x + 7$

5. Let α and 1α be the zeroes of P(x).

$P(a) = ax^2 + bx + c$ …(given)

Product of zeroes = $\frac{c}{a}$

$\Rightarrow \alpha \times \frac{1}{\alpha} = \frac{c}{a}$

$\Rightarrow 1 = \frac{c}{a}$

$\Rightarrow a = c$ (Required condition)

Coefficient of x^2 = Constant term

SECTION B

6. We have, $2x^2 - 5x - 3 = 0$

$= 2x^2 - 6x + x - 3$

$= 2x(x - 3) + 1(x - 3)$

$= (x - 3)(2x + 1)$

Zeroes are:

$x - 3 = 0$ or $2x + 1 = 0$

$\Rightarrow x = 3$ or $x = -\frac{1}{2}$

Since the zeroes of the required polynomial are double of the given polynomial.

Zeroes of the required polynomial are:

$3 \times 2, (-\frac{1}{2} \times 2)$, i.e., 6, -1

Sum of zeroes, $S = 6 + (-1) = 5$

Product of zeroes, $P = 6 \times (-1) = -6$

Quadratic polynomial is x^2 – (Sum)x + Product

$\Rightarrow x^2 - 5x - 6$ …(i)

Comparing (i) with $x^2 + px + q$

$p = -5, q = -6$

7. $p(x) = 2x^2 - x - 6$ …[Given]
$= 2x^2 - 4x + 3x - 6$
$= 2x(x - 2) + 3(x - 2)$
$= (x - 2)(2x + 3)$
Zeroes are:
$x - 2 = 0$ or $2x + 3 = 0$
$x = 2$ or $x = -\frac{3}{2}$
Verification:
Here a = 2, b = -1, c = -6

Sum of zeroes $= 2 + \left(\frac{-3}{2}\right) = \frac{4-3}{2} = \frac{1}{2}$

$= \frac{1}{2} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-b}{a}$

Product of zeroes $= 2 \times \left(\frac{-3}{2}\right) = \frac{-6}{2}$

$= \frac{-6}{2} = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$

∴ **Relationship holds.**

8. $f(x) = x^2 - x - 4$ i.e.
If α and β are the zeroes

$\therefore \alpha + \beta = \frac{1}{1} = 1$

$\alpha . \beta = \frac{-4}{1} = -4$

So,

$$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha + \beta}{\alpha\beta} - \alpha\beta$$

$$= \frac{1}{-4} - (-4)$$

$$= -\frac{1}{4} + 4$$

$$= \frac{15}{4}$$

9.

$\alpha+\beta=-p$

$\alpha\beta=45$

$(\alpha-\beta)^2=144$

$\Rightarrow \alpha^2+\beta^2-2\alpha\beta$

$\Rightarrow (\alpha+\beta)^2-4\alpha\beta=144$

$\Rightarrow (-p)^2-4\times 45=144$

$\Rightarrow p^2=144+180$

$\Rightarrow p=\pm 18$

10. Sum of the zeros = half of product

$\Rightarrow (k+6)=\frac{1}{2}\left[2(2k+1)\right]$

$\Rightarrow k+6=2k+1$

$\Rightarrow k=5$

SECTION C

11.

$f(x)=x^2-p(x+1)-c$

$=x^2-px-(p+c)$

$\therefore \alpha+\beta=p \text{ and } \alpha\beta=-(p+c)$

$Now\ (\alpha+1)(\beta+1)=\alpha\beta+(\alpha+\beta)+1$

$=-p-c+p+1$

$=1-c$

12. Let the zeros are 7p and 6p.

$3x^2-k+14$

$\therefore 7p+6p=\frac{-(-k)}{3}=\frac{k}{3}$

and $7p\times 6p=\frac{14}{3}$

$\Rightarrow 42p^2=\frac{14}{3}$

$p=3$

$\Rightarrow 39p=k$

$\therefore k=39\times 3$

$\therefore k=117$

13. $p(x)=x^4+10x^3+25x^2+15x+k$

$\therefore\ (x+7)$ is the factor.

$\therefore\ p(-7)=0$

$or\ (-7)^4+10(-7)^3+25(-7)^2+15(-7)+k=0$

$2401-3430+1225-105+k=0$

$k=91$

14.

$f(x)=x^2+px+q,$ if α and β are zeros

$\therefore\ \alpha+\beta=\text{-}p\ and\ \alpha\beta=q$

If zeros are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$=(-p)^2-4q$

$(\alpha-\beta)^2=-p^2-4q$

Now sum of zeros

$(\alpha+\beta)^2+(\alpha-\beta)^2=(-p)^2+(p^2-4q)$

$=2p^2-4q$

Pr oduct of zeros

$(\alpha+\beta)^2(\alpha-\beta)^2=(-p)^2+(p^2-4q)$

$=4p^4-4p^2q$

$\therefore$ required polynomial is

$x^2-(\text{sum of zeros})\,x+$ product of zeros

$=x^2-(2p^2-4q)x+4p^4-4p^2q$

$=x^2-2p^2x-4qx+p^4-4p^2q$

15.

$p(x)=4\sqrt{3}x^2+5x-2\sqrt{3}$

$=4\sqrt{3}x^2+8x-3x-2\sqrt{3}$

$=4x(\sqrt{3}x+2)-\sqrt{3}(\sqrt{3}x+2)$

$=(4x-\sqrt{3})(\sqrt{3}x+2)$

$\therefore$ zeros are $4x-\sqrt{3}=0$ and $\sqrt{3}x+2=0$

$\Rightarrow\ x=\frac{\sqrt{3}}{4}$ and $x=-\frac{2}{\sqrt{3}}$

$$\text{Sum of zeros} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \left[\frac{\sqrt{3}}{4} + \frac{(-2)}{\sqrt{3}}\right] = \frac{-5}{4\sqrt{3}}$$

$$\text{Product of zeros} = \frac{\text{Constant term}}{\text{Cofficient of } x^2}$$

$$= \frac{-2\sqrt{3}}{4\sqrt{3}} = \frac{-1}{2}$$

SECTION D

16. (i) (b): Graph of a quadratic polynomial is a parabolic in shape.
(ii) (c): Since the graph of the polynomial cuts the
x-axis at (-6,0) and (6, 0). So, the zeroes of the polynomial are -6 and 6.
∴∴ Required polynomial is $p(x) = x^2 - (-6 + 6)x + (-6)(6) = x^2 - 36$
(iii) (c) : We have, $p(x) = x^2 - 36$
Now, p(6) = 62 - 36 = 36 - 36 = 0
(iv) (b): Let $f(x) = x^2 + 2x - 3$. Then,
Sum of zeroes $=-2$
(v) (d): The given polynomial is $at^2 + 5t + 3a$ Given, sum of zeroes = product of zeroes.
$-\frac{5}{a} = \frac{3a}{a}$

$-\frac{5}{3} = a$

17. (i) (b): Given, α and β are the zeroes of
$p(x) = = x^2 - 24x + 128$
P(x) = 0
$x^2 - 8x - 16x + 128 = 0$
⇒x(x-8)-16(x-8) =0
⇒(x−8)(x−16)=0⇒x=8 or x=16
α=8,β=16
(ii) (c) : α+β+αβ=8+16+(8)(16)=24+128=152
(iii) (d) : p(2)=22−24(2)+128=4−48+128=84
(iv) (a): Since , α and β are the zeroes of
$p(x) = = x^2 + x - 2$
∴α+β=−1 and αβ=−2
Now, $\frac{1}{\alpha} + \frac{1}{\beta}$
$= \frac{\beta+\alpha}{\alpha\beta} = \frac{-1}{-2} = \frac{1}{2}$
(v) (c): Sum of zeroes = $\frac{-2}{k}$

Product of zeroes $=\frac{3k}{k} = 3$

According to question, we have $-\frac{-2}{k}=3$

$\frac{-2}{3}=k$

18. $p(x) = x^3 - 2x^2 + kx + 5$,

When x – 2,

$p(2) = (2)^3 - 2(2)^2 + k(2) + 5$

$\Rightarrow 11 = 8 - 8 + 2k + 5$

$\Rightarrow 11 - 5 = 2k$

$\Rightarrow 6 = 2k$

$\Rightarrow k = 3$

Let $q(x) = x^3 + kx^2 + 3x + 1$

$= x^3 + 3x^2 + 3x + 1$

$= x^3 + 1 + 3x^2 + 3x$

$= (x)^3 + (1)^3 + 3x(x + 1)$

$= (x + 1)^3$

$= (x + 1)(x + 1)(x + 1)$ …[$\because a^3 + b^3 + 3ab(a + b) = (a + b)^3$]

All zeroes are:

$x + 1 = 0 \Rightarrow x = -1$

$x + 1 = 0 \Rightarrow x = -1$

$x + 1 = 0 \Rightarrow x = -1$

Hence zeroes are -1, -1 and -1

19. We have, $p(x) = kx^2 + 4x + 4$

Here a = k, b = 4, c = 4

Sum of zeroes, $\alpha + \beta = \frac{-b}{a} = \frac{-4}{k}$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{4}{k}$

$\alpha^2 + \beta^2 = 24$ ...[Given

$(\alpha + \beta)^2 - 2\alpha\beta = 24$

$\left(\frac{-4}{k}\right)^2 - 2\left(\frac{4}{k}\right) = 24$

$\Rightarrow \frac{16}{k^2} - \frac{8}{k} = 24 \quad \Rightarrow \quad \frac{16 - 8k}{k^2} = \frac{24}{1}$

$\Rightarrow 24k^2 = 16 - 8k$

$\Rightarrow 24k^2 + 8k - 16 = 0$

$\Rightarrow 3k^2 + k - 2 = 0$ …[Dividing both sides by 8]

$\Rightarrow 3k^2 + 3k - 2k - 2 = 0$

$\Rightarrow 3k(k + 1) - 2(k + 1) = 0$

$\Rightarrow (k + 1)(3k - 2) = 0$

$\Rightarrow k + 1 = 0$ or $3k - 2 = 0$

$\Rightarrow k = -1$ or $k = \frac{2}{3}$

20. Given polynomial is $p(x) = 2x^2 + 5x + k$
Here $a = 2$, $b = 5$, $c = k$

$$\alpha + \beta = \frac{-b}{a} = \frac{-5}{2}; \qquad \alpha\beta = \frac{c}{a} = \frac{k}{2}$$

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4} \qquad \text{...[Given}$$

$$(\alpha + \beta)^2 - 2\alpha\beta + \alpha\beta = \frac{21}{4}$$

$$(\alpha + \beta)^2 - \alpha\beta = \frac{21}{4} \quad \Rightarrow \quad \left(\frac{-5}{2}\right)^2 - \left(\frac{k}{2}\right) = \frac{21}{4}$$

$$\frac{-k}{2} = \frac{21}{4} - \frac{25}{4} = \frac{-4}{4} \quad \Rightarrow \quad \frac{-k}{2} = -1 \; \therefore \; \boldsymbol{k = 2}$$

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Points to remember

If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then then following situations can arise:

i. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: in this case, the pair of linear equations is consistent.

ii. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$: in this case, the pair of linear equations is inconsistent.

iii. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$: in this case, the pair of linear equations is dependent and consistent.

(1 mark questions)

1. The pair of equations has infinitely many solutions if;

   $3x + 4y = k$

   $9x + 12y = 6$

   (a) k=2 (b) k=6 (c) k=6 (d) k=3

2. The pair of equations

   $3x + 4y = 18$

   $4x + 16/3y = 24$ has

   (a) no solutions (b) a unique solution (c) infinitely many solutions

   (d) cannot say anything

3. If $2x + 3y = 0$ and $4x - 3y = 0$, then $x + y$ equals
   (a) 0 (b) -1 (c) 1 (d) 2

4. The pair of equations x=a and y=b graphically represent lines which are
   (a) parallel (b) intersecting at (a,b) (c) coincident

   (d) intersecting at (b, a)

5. If the system of equations $kx - 5 = 2$, $6x + 2y = 7$ has no solutions, then k=.....

   (a) -10 (b) -5 (c) -6 (d) -15

(2 marks questions)

1. Given the linear equations 2x + 3y – 8 == 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is intersecting lines.

2. The cost of 2kg apples and 1kg of grapes on a day was found to be 160 rupees. After a month, the cost of 4kg of apples and 2kg of grapes is 300 rupees. Represent the situation algebraically.

3. Solve the following pair of linear equations by substitution method:

   $2x + y - 6 = 0$

   $3x + 2y - 11 = 0$

4. Solve the following pair of linear equation by the elimination method:

   $x + y = 14$

   $x - y = 4$

5. The coach of a cricket team buys 3 bats and 6 balls for 3000 rupees. Later, she buys another bat and 3 more balls of the same kind for 1300 rupees. Represent this situation algebraically.

(3 marks questions)

1. Solve the following systems of equations:
   $3x - 7y + 10 = 0$
   $y - 2x - 3 = 0$

2. Solve the following systems of equations:
   $\frac{x}{3} + \frac{y}{4} = 11$
   $5\frac{x}{6} - \frac{y}{3} = -7$

3. In the following system of equations determine whether the system has a unique solution:
   $2x + 3y = 7$
   $6x + 5y = 11$

4. Find the value of k for which the following system of equations has a unique solution:

   $kx + 2y = 5$

   $3x + y = 1$

5. For what value of k, the following system of equations will represent the coincident lines?
   $x + 2y + 7 = 0$,
   $2x + ky + 14 = 0$

(5 marks questions)

1. 4 chairs and 3 tables cost Rs.2100 and 5 chairs and 2 tables cost Rs.1750.Find the cost of a chair and a table separately.
2. Sum of two numbers is 35 and their difference is 13.Find the numbers.
3. A fraction becomes 4/5 if 1 is added to both numerator and denominator. If 5 is subtracted from both numerator and denominator the fraction becomes ½.What is the fraction?
4. If twice the son's age in years is added to the father's age the sum is 70.But if twice the father's age is added to the son's age the sum is 95.Find the ages of father and son.

5. Solve the system of equations graphically:
   $x + y = 3$
   $2x + 5y = 12$

Answer key

(1 mark questions)

(1) k=2

(2) Infinitely any solutions

(3) 0

(4) Intersecting at (a,b)

(5) -15

(2 marks questions)

(1) $x + 2y – 4$, or any other correct answer

(2) $2x + y = 160$, $4x + 2y = 300$

(3) $x = 1$, $y = 4$

(4) $x = 9$, $y = 5$

(5) $3x + 6y = 3900$, $x + 3y = 1300$

(3 marks questions)

(1) $x = -1$, $y=1$

(2) $x = 6$, $y = 36$

(3) yes, the system has a unique solution

(4) k = 6

(5) k = 4

(5 marks questions)

(1) 150 ,500

(2) 24 and 11

(3) 7/9

(4) 40 and 15

(5) x=1 and y=2

QUADRATIC EQUATION

- A quadratic equation is represented as $ax^2 + bx + c = 0$, where a, b, c are real numbers and a is not equal to 0.
- A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.
- The roots of a quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

- A quadratic equation has:

  Two distinct real roots if $b^2 - 4ac > 0$;

  Two equal roots if $b^2 - 4ac = 0$;

  No real roots if $b^2 - 4ac < 0$.

SECTION –A (1 mark)

1. Find the nature of the roots of the Quadratic Equation $2x^2 - 4x + 3 = 0$
   a)Real roots b) equal roots c) no real roots d) none
2. Find the value of k for which the roots are real and equal in the equation $3x^2 - 5x + 2k = 0$.
   a)5/4 b) 25/24 c) 24/25 d) none
3. Determine k for which the the equation has real roots $2x^2 - 5x - k = 0$
   a)$k \geq -25/8$ b) $k \leq 25/8$ c) $k=25/8$ d) none
4. Which one of the following is not a quadratic equation
   a)$2x^2 + 3x + p = 0$ b)$x^2 - 3x = 0$. c) $(x + 5)^2 = 2(5x - 3)$ d) $3x - \sqrt{5x} + 5 = 0$
5. The solution of the equation $x^2 - 3x + 2 = 0$. are
   a) $x=2, x=1$ b) $x=2, x\neq-1$ c) $x=2, x=-1$
   d) $x\neq2, x=-1$

SECTION –B (2 marks)

6. For what value of p the equation $2x^2 + 3x + p = 0$ will have real roots.
7. Find the sum of the roots of the quadratic equation $3x^2 - 9x + 5 = 0$.
8. Find the roots of the quadratic equation $x^2 - 3x = 0$.
9. What value of k does the equation $2x^2 - kx + k = 0$ has equal roots?
10. Write the discriminant of the equation $(x + 5)^2 = 2(5x - 3)$

SECTION –C (3 marks)

11. The sum of a number and its reciprocal is 5/2. Find the numbers.
12. Find the roots of the quadratic equation $4x^2$- 4px + $(p^2 - q^2)$ =0.
13. If one root of the equation 2 x^2+ kx + 4 = 0 is 2, then find its other root.
14. If 2 is a root of the $3x^2$ + px - 8 = 0 and the quadratic equation $4x^2$ - 2px + k = 0 has equal roots, find the value of k.
15. The product of two consecutive natural numbers is 72. Find the numbers.

SECTION –D (4 marks)

16. The sum of the ages of a son and his father is 35 years and the product of their ages is 150 years , find their ages.
17. A passenger train takes 2 hours less for a journey of 300km if its speed is increased by 5 km/hr from its usual speed. Find the usual speed of the train?
18. Speed of a boat in still water is 11km/hr. It can go 12 km upstream and return downstream to the original point in 2hrs 45min. Find the speed of the stream.
19. Solve for x: $\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}$, $x \neq -1, -1/5, -4$
20. Solve for x: $\frac{x-3}{x-4} + \frac{x-5}{x-6} = \frac{10}{3}$, $x \neq 4, 6$

ANSWERS

1. NO REAL ROOTS
2. K=25/24
3. $k \geq -25/8$
4. $3x - \sqrt{5}x + 5 = 0$
5. x=2, x=1
6. $p \leq 9/8$
7. 3
8. X=0, x=3
9. K=0,8
10. D=-124
11. x=2 and 1/2
12. $\frac{p \pm q}{2}$
13. Other root is 1
14. K=1

15. 8,9
16. Son=5yrs, father=30yrs
17. Speed= 25km/hr
18. Speed=5km/hr
19. x=-11/17 , 1
20. x=7, 9/2

ARITHMETIC PROGRESSION

An arithmetic progression (AP) is a progression in which the difference between two consecutive terms is constant.

In arithmetic progression, the first term is represented by the letter "a", last term is represented by "l", the common difference between two terms is represented by "d" and the number of terms is represented by the letter "n".

Thus, the standard form of the arithmetic progression is given by the formula,

a, a + d, a + 2d, a + 3d, a + 4d, ….

Common Difference

The difference between two consecutive terms in an AP, (*which is constant*) is the "common difference"(d) of an A.P. In the progression: *2, 5, 8, 11, 14* …the common difference is 3.

As it is the difference between any two consecutive terms, for any A.P, if the common difference is:

- positive, the AP is increasing.
- zero, the AP is constant.
- negative, the A.P is decreasing.

The formula to find the common difference between the two terms is given as:

Common difference, $d = (a_n - a_{n-1})$

Where, a_n represents the nth term of a sequence

a_{n-1} represents the previous term. i.e., (n-1)th term of a sequence.

The nth term of an AP

The nth term of an A.P is given by

$$a_n = a + (n-1)d$$

where **a** is the first term, **d** is a common difference and **n** is the number of terms.

Sum of Terms in an AP

The formula for the sum to n terms of an AP

The sum to n terms of an A.P is given by:

$$S_n=\frac{n}{2}(2a+(n-1)d$$

Where **a** is the first term, **d** is the common difference and **n** is the number of terms.

The sum of n terms of an A.P is also given by

$$S_n=\frac{n}{2}(a+l)$$

Where **a** is the first term, **l** is the last term of the A.P. and **n** is the number of terms.

SECTION – A (1 Mark)

1.Which term of the AP: 21,42,63, …………. is 210

(a) 9th (b) 10th (c) 11th (d) 12th

2. The 11th term from the last term of the AP : 10,7,4, …………. – 62 is

(a) 32 (b) 64 (c) – 64 (d) – 32

3. The 21st term of an AP whose first two terms are – 3 and 4 is

(a) 17 (b) 137 (c) 143 (d) 38

4. The second term of an AP is 13 and 5th term is 25,then the 7th term is

(a) 33 (b) 29 (c) 37 (d) 35

5. The sum of first 100 multiples of 3 is

(a) 30300 (b) 15150 (c) 300 (d) None of these

SECTION – B (2 Marks)

6. If 2p , p + 10 , 3p + 2 are in AP, find the value of p

7. Find the sum of the first 15 multiples of 8.

8. In an AP , the nth term of an AP is given by an = 3 – 4 n , find the first term and common difference.

9. How many two-digit numbers are divisible by 3?

10. Which term of the AP : 3 , 8, 13 , 18 …... is 78 ?

SECTION – C (3 Marks)

11. The sum of the 5th term and 7th term of an AP is 52 and the 10th term is 46.Find the first three terms of the AP.

12. If the 5th term of an AP is zero, show that its 33rd is four times its 12th term.

13. A man receives Rs 60 for the first week and Rs 3 more each week than the preceding week. How much does he earn by the 20th week?

14. How many terms of the AP : 18,16,14 …... should be taken so that their sum is zero.

15. The first term of an AP is – 5 and the last term is 45.If the sum of the terms of the AP is 120,find the number of terms and common difference.

SECTION – D (5 Marks)

16. 390 plants are to be planted in a garden in a number of rows. There are 40 plants in the first row,38 plants in the second row,36 plants in the third row and so on. Each next row has two plants less than those in the previous row. In how many rows the 390 plants are planted? Also find the number of plants in the last row.

17. A manufacturer of TV sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find

(a) the production in the first year

(b) the production in the 10th year

(c) the total production in 7 years.

18. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows. Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day etc. The penalty for each succeeding day being Rs 50 more than the preceding day. How much money does the contractor have to pay as penalty if he has delayed the work by 30 days?

19. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees ,that each section of each class will plant, will be same as the class, in which they are studying, a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII.There are four sections of each class. Based on this, answer the following.

(a) Find the number of trees planted by students of Class II.

(b)The students of which class have planted 40 trees?

(c) Find the total number of trees planted by the students of all classes.

20. The sum of three numbers in an AP is 27 and their product is 405.Find the numbers.

Marking Scheme

SECTION - A

1.(b) 10th

2. (d) – 32

3. (b) 137

4. (a) 33

5. (b) 15150

SECTION – B

6. 2p, p + 10, 3p + 2 are in AP

P + 10 – 2p = 3p + 2 – (p + 10)

10 – p = 3p + 2 – p – 10

10 – p = 2p – 8

18 = 3p

P = 6

7. The first 15 multiples of 8 are 8,16,24,32……...120

(a + an)

(8 + 120)

= 960

8. an = 3 – 4n

a1= 3 – 4 × 1 = -1

a2 = 3 – 4 ×2 = - 5

d = - 5 – (- 1)

d = - 4

9. Two-digit numbers divisible by 3 are 12 , 15 , 18 , …. 99

99 = 12 + (n-1)3

99 – 12 = (n-1)3

= n – 1

29 = n – 1

n = 30

So 30 two digit numbers are divisible by b3

10. 3,8,13,18 …… 78 is an AP

78 = 3 + (n-1)×5

= n – 1

N = 16

So 16th term of the AP is 78

SECTION – C

11. a5 + a7 = 52

a10 = 46

a + 4d + a +6d = 52

a + 9d = 46

Solving d = 5 and a = 1

So the AP is 1 , 6 , 11….

12. a5 = 0

a + 4d = 0

a = - 4d

a33 = a + 32d

= - 4d + 32d = 28d ………..(1)

a12 = a + 11d

= - 4d + 11d = 7d ………..(2)

From (1) and (2) the 33rd term is 4 times the 12th term.

13. 60, 63, 66 ……. Forms an AP

a20 = a + 19d

= 60 + 19 ×3

= Rs 117

14. 18,16,14,….. is an AP

[2 a + (n- 1)d]

[36 + (n-1)×(-2)]

n(36-2n +2) = 0

Solving n = 19

15. a = - 5 , an = 45

[a + an]

-5 + 45)

Solving n = 6 and d = 10

SECTION – D

16. The number of plants are 40 , 38 ,36, ……

[2a + (n-1)d]

[80 + (n-1)(-2)]

Simplifying n2 – 41n +390 = 0

Solving n = 15 or n = 26(rejected)

a15 = 40 + 14×(-2)

a15 = 12

Number of rows = 15

Number of plants in the last row = 12

17. a3 = 600 , a7 = 700

a+ 6d = 700…….(1)

a + 2d = 600…….(2)

Solving a=550 and d = 25

a10 = 775

S7 = 4375

18. The penalty for 30 days are as follows

200,250,300,350 …….

[2a + (n-1)d]

$S_{30} = 15(400 + 29 \times 50)$

$= 15(400 + 1450)$

$= 15 \times 1850$

$= 27750$

Total penalty for 30 days = Rs 27750

19. (a) 8

(b) Class X

(c) 312

20. Let the consecutive terms be a – d , a and a + d

$a - d + a + a + d = 27$

$3a = 27$

$a = 9$

$a(a-d)(a+d) = 405$

$9(81 - d2) = 405$

$81 - d2 = 45$

$36 = d2$

$d = \pm 6$

The terms are 3 , 9 ,15

TRIANGLES

IMPORTANT CONCEPTS

Similar figures :Two figures having same shape (size may or may not same) are called similar figures

Eg: (1) All Circles are similar

(2) Equilateral triangles are similar

Similar triangles: Two triangles are said to be similar if

(a) Corresponding angles of both the triangles are equal
(b) Corresponding sides of both the triangles are in proportion .

Basic Proportionality Theorem(Thales theorem) : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Criterion of Similarity

- In two triangles, if the corresponding angles are equal, then the corresponding sides are in proportion, then the triangles are similar (AAA similarity criterion)
- If the corresponding sides of any two triangles are proportional, then the corresponding angles are equal and the two triangles are similar (SSS similarity criterion)
- If one angle of a triangle is equal to one angle of the another triangle and the corresponding sides including these angle are proportional, then the triangles are similar (SAS similarity criterion)

SECTION :A MCQ(1 Mark each)

Q 1. **If ΔABC is similar to ΔDEF such that 2 AB = DE and BC = 8 cm then EF is equal to.**
a. 12 cm
b. 4 cm
c. 16 cm
d. 8 cm

Q 2. **D and E are the midpoints of side AB and AC of a triangle ABC, respectively and BC = 6 cm. If DE || BC, then the length (in cm) of DE is:**

(a) 2.5

(b) 3

(c) 5

(d) 6

Q3. **If ABC and EDF are two triangles and AB/DE=BC/FD, then the two triangles are similar if**

(a) ∠A=∠F

(b) ∠B=∠D

(c) ∠A=∠D

(d) ∠B=∠E

Q4. **In triangles ABC and DEF, ∠B = ∠E, ∠F = ∠C and AB = 3 DE. Then, the two triangles are**

(a) congruent but not similar

(b) similar but not congruent

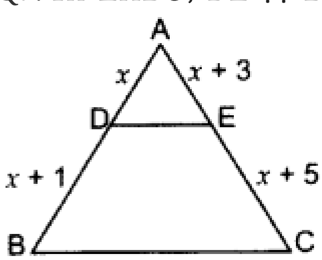
(c) neither congruent nor similar

(d) congruent as well as similar

Q5. **In ΔLMN, ∠L = 50° and ∠N = 60°, If ΔLMN ~ ΔPQR, then find ∠Q**
a. 50°
b. 70°
c. 60°
d. 40°

SECTION :B(2 Marks each)

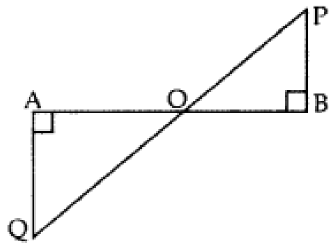
Q6. If ΔABC ~ ΔRPQ, AB = 3 cm, BC = 5 cm, AC = 6 cm, RP = 6 cm and PQ = 10, then find QR.
Q7. In ΔABC, DE || BC, find the value of x.



Q8. . X and Y are points on the sides AB and AC respectively of a triangle ABC such that **AX/AB**=1/4, AY = 2 cm and YC = 6 cm. Find whether XY || BC or not.

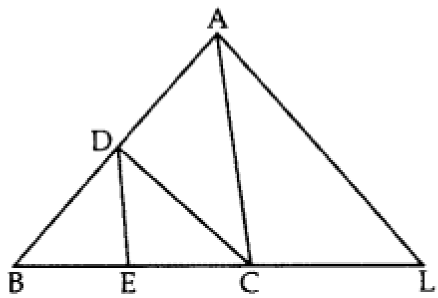
Q9. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE is similar to ΔCFB.

Q10. In the given figure, QA ⊥ AB and PB ⊥ AB. If AO = 20 cm, BO = 12 cm, PB = 18 cm, find AQ.



SECTION :C(3 Marks each)

Q11. In the given figure, CD || LA and DE || AC. Find the length of CL if BE = 4 cm and EC = 2 cm.

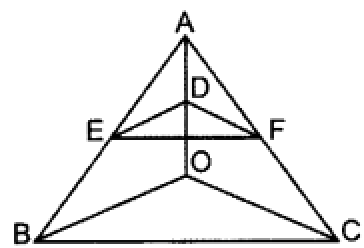


Q12. If a line segment intersects sides AB and AC of a ΔABC at D and E respectively and is parallel to BC, prove that $\frac{AD}{AB} = \frac{AE}{AC}$

Q13. In a ΔABC, DE || BC with D on AB and E on AC. If $\frac{AD}{DB} = \frac{3}{4}$, find $\frac{BC}{DE}$

Q14. A vertical pole of length 8 m casts a shadow 6 cm long on the ground and at the same time a tower casts a shadow 30 m long. Find the height of tower

Q15. In the figure, if DE || OB and EF || BC, then prove that DF || OC.



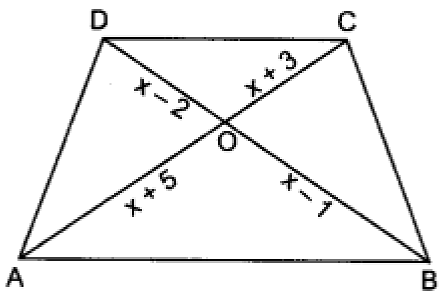
SECTION :D (5 Marks each)

Q16. If sides AB, BC and median AD of AABC are proportional to the corresponding sides PQ, QR and median PM of PQR, show that ΔABC ~ ΔPQR

Q17. (a)"If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio". Prove (b). In ΔDEW, AB || EW. If AD = 4 cm, DE = 12 cm and DW = 24 cm, then find the value of DB.

Q18. (a) The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ Show that ABCD is a trapezium.

(b). In the given figure, if AB || DC, find the value of x.



Q19 (a). In the given Fig, $PS/SQ = PT/TR$ and ∠PST = ∠PRQ. Prove that PQR is an isosceles triangle.

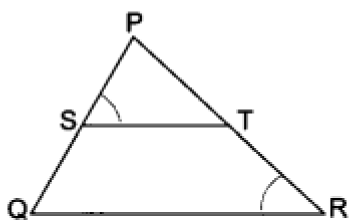
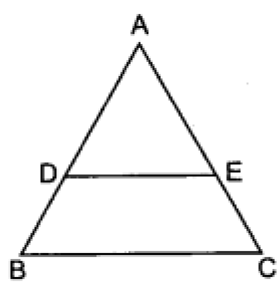


Fig. 7.29

(b) In figure, DE|| BC and BD = CE. Prove that ΔABC is an isosceles triangle.



Q20. In ΔABC, if ∠ADE = ∠B, then prove that $\Delta ADE \sim \Delta ABC$. Also, if AD = 7.6 cm, AE = 7.cm, BE = 4.2 cm and BC = 8.4 cm, then find DE.

ANSWERS

SECTION :A

1) (c) 16 cm 2) (b) 3 3) (b) <B = <D 4) (b) Similar not congruent 5) (b) 70^0

SECTION :B (2 Marks each)

Q6. $\Delta ABC \sim \Delta RPQ$ ...[Given

$\therefore \frac{AB}{RP} = \frac{BC}{PQ} = \frac{AC}{RQ}$ ...[In ~ Δs corresponding sides are proportional

$\Rightarrow \frac{3}{6} = \frac{5}{10} = \frac{6}{QR} \Rightarrow \frac{1}{2} = \frac{6}{QR}$

$\therefore$ QR = 12 cm

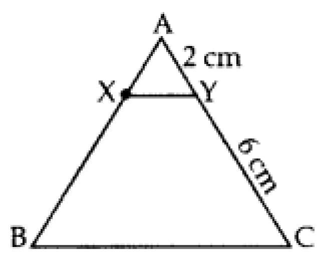
Q7. In ΔABC, DE || BC ...[Given

$\frac{AD}{BD} = \frac{AE}{EC}$ ...[Thales' theorem

$\frac{x}{x+1} = \frac{x+3}{x+5}$

$x(x + 5) = (x + 3)(x + 1)$
$x^2 + 5x = x^2 + 3x + x + 3$
$x^2 + 5x - x^2 - 3x - x = 3$
$\therefore$ x = 3 cm

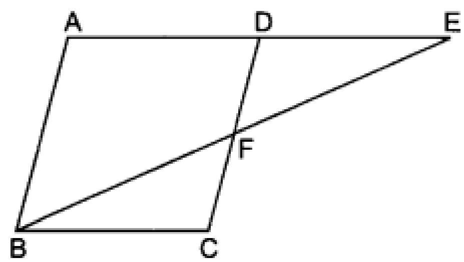
Q8. Given: $AX/AB = 1/4$



AX = 1K, AB = 4K
$\therefore$ BX = AB – AX
= 4K – 1K = 3K

$\frac{AX}{XB} = \frac{1K}{3K} = \frac{1}{3}$

$\frac{AY}{YC} = \frac{2}{6} = \frac{1}{3} \quad \therefore \quad \frac{AX}{XB} = \frac{AY}{YC} = \frac{1}{3}$



Q9.

In ΔABE and ΔCFB, we have

$\angle AEB = \angle CBF$ (Alternate angles)

$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\therefore \Delta ABE \sim \Delta CFB$ (By AA criterion of similarity)

Q10. In ΔOAQ and ΔOBP,

$\angle OAQ = \angle OBP$ … [Each 90°

$\angle AOQ = \angle BOP$ … [vertically opposite angles

$\therefore \quad \Delta OAQ \sim \Delta OBP$ **...[By AA corollary**

$\frac{AO}{BO} = \frac{AQ}{PB}$ **...[$\because$ sides are proportional**

$\frac{20}{12} = \frac{AQ}{18} \quad \Rightarrow \quad AQ = \frac{18 \times 20}{12}$

AQ = 30 cm

SECTION :C

Q11. In ΔABL, CD || LA

$\frac{BD}{DA} = \frac{BC}{CL}$ ...(*i*) **[Thales' theorem**

In ΔABC, DE || AC

$\frac{BD}{DA} = \frac{BE}{EC}$ ...(*ii*) **[Thales' theorem**

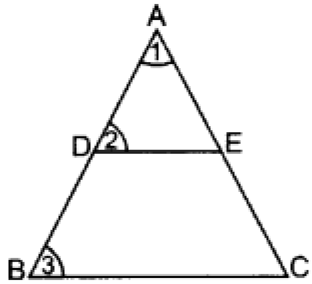
From (*i*) and (*ii*), we get

$\frac{BC}{CL} = \frac{BE}{EC} \quad \Rightarrow \quad \frac{BE + EC}{CL} = \frac{BE}{EC}$

$\Rightarrow \frac{4 + 2}{CL} = \frac{4}{2}$ **...[BE = 4 cm, EC = 2 cm (Given)**

$\Rightarrow 2CL = 6 \quad \therefore$ **CL = 3 cm**

Q12. Given. In ΔABC, DE || BC



To prove. AD/AB=AE/AC

Proof.

In ΔADE and ΔABC

∠1 = ∠1 … Common

∠2 = ∠3 ⋯ [Corresponding angles

ΔADE ~ ΔABC ⋯[AA similarity

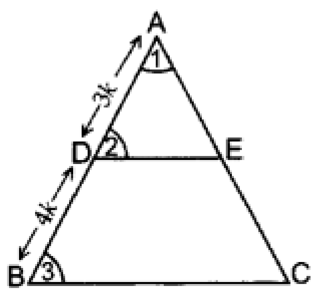
∴ **AD/AB=AE/AC**

⋯[In ~Δs corresponding sides are proportional]

Q13. Given: In a ΔABC, DE || BC with D on AB and E on AC and *AD/DB*=3/4

To find: BCDE

Proof. Let AD = 3k,



DB = 4k

∴ AB = 3k + 4k = 7k

In ΔADE and ΔABC,

∠1 = ∠1 …[Common

∠2 = ∠3 … [Corresponding angles

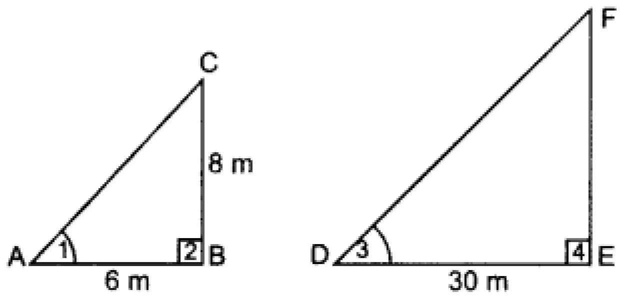
∴ ΔADE ~ ΔABC ⋯[AA similarity]

AD/AB = DE/BC [In ~Δs corresponding sides are proportional]

3k/7k = DE/BC

Therefore, BC/DE = 7/3

Q14.



Let BC be the pole and EF be the tower Shadow AB = 6 m and DE = 30 m.

In ΔABC and ΔDEF,

$\angle 2 = \angle 4$ … [Each 90°

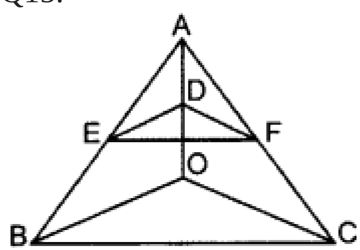
$\angle 1 = \angle 3$ … [Sun's angle of elevation at the same time]

$\Delta ABC \sim \Delta DEF$ ⋯[AA similarity]

$AB/DE = BC/EF$ … [In similar triangles corresponding sides are proportional]

$\Rightarrow 6/30 = 8/EF \therefore EF = 40$ m

Q15.



Solution:

Given. In ΔABC, DE || OB and EF || BC

To prove. DF || OC

Proof. In ΔAOB, DE || OB ⋯ [Given

$$\frac{AE}{EB} = \frac{AF}{FC}$$ ...(*ii*) **[Thales' theorem**

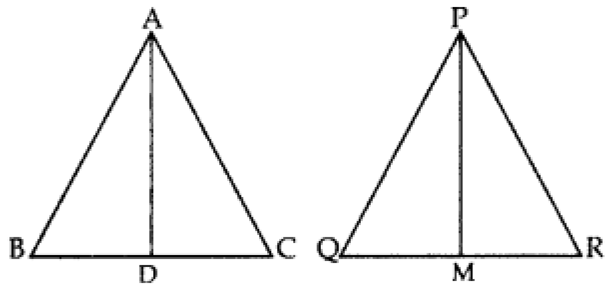
From (*i*) and (*ii*), $\frac{AD}{DO} = \frac{AF}{FC}$

$\therefore$ In ΔAOC, **DF || OC**

...[By converse of Thales' theorem

Section :D (5 Marks each)

Q 16.Solution:



Given: $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

To prove: $\Delta ABC \sim \Delta PQR$

Proof: In ΔABD and ΔPQM

Given: $\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QR} = \frac{AD}{PM}$

$\because$ Sides of Δs are proportional.

$\therefore$ $\Delta ABD \sim \Delta PQM$ **...[SSS similarity**

$\angle ABD = \angle PQM$ **...[c.p.c.t.**

$\Rightarrow$ $\angle ABC = \angle PQR$

In ΔABC and ΔPQR,

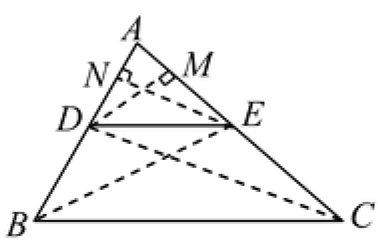
$\frac{AB}{PQ} = \frac{BC}{QR}$ (*i*) **...[Given**

$\Rightarrow$ $\angle ABC = \angle PQR$ ...(*ii*)

From (*i*) & (*ii*),

ΔABC ~ ΔPOR **...[SAS similarity**

Q17.(a) Given: A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.



To prove: $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction: Join BE and CD and draw DM ⊥ AC and EN ⊥ AB.

Proof: area of Δ ADE $(=\frac{1}{2}\text{base}\times\text{height})=\frac{1}{2}AD\times EN.$

(Taking AD as base)

So, $ar(BDE)=\frac{1}{2}DB\times EN,$ [The area of Δ ADE is denoted as ar (ADE)].

Similarly, $ar(BDE)=\frac{1}{2}DB\times EN,$

$ar(ADE)=\frac{1}{2}AE\times DM$ and $ar(DEC)=\frac{1}{2}EC\times DM.$ (Taking AE as base)

∴ $\frac{ar(ADE)}{ar(BDE)}=\frac{\frac{1}{2}AD\times EN}{\frac{1}{2}DB\times EN}=\frac{AD}{DB}$ …(i)

and $\frac{ar(ADE)}{ar(DEC)}=\frac{\frac{1}{2}AE\times DM}{\frac{1}{2}EC\times DM}=\frac{AE}{EC}$ …(ii)

$ar(BDE)=ar(DEC)$. … (iii)

[Δ BDE and DEC are on the same base DE and between the same parallels BC and DE.]

Therefore, from (i), (ii) and (iii), we have:

$\frac{AD}{DB}=\frac{AE}{EC}.$.

(b))
Let BD = x cm
then BW = (24 – x) cm, AE = 12 – 4 = 8 cm

In $\triangle DEW$, AB || EW

$$\frac{AD}{AE} = \frac{BD}{BW}$$

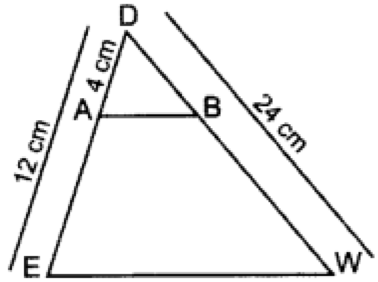
...[Thales' Theorem

$$\frac{4}{8} = \frac{x}{24 - x}$$

$$8x = 96 - 4x$$

$$\Rightarrow \quad 12x = 96$$

$$\Rightarrow \quad x = \frac{96}{12} = 8 \text{ cm} \qquad \therefore \quad \mathbf{DB = 8\ cm}$$



Q18. Given: Quadrilateral ABCD in which
AC and BD intersect each other at 0.
Such that $AO/BO=CO/DO$
To prove: ABCD is a trapezium
Const.: From O, draw OE || AB.

Solution:

$$\frac{AO}{BO} = \frac{CO}{DO} \qquad \text{(Given)}$$

$$\Rightarrow \quad \frac{AO}{CO} = \frac{BO}{DO} \qquad ...(i)$$

In ΔABD, $EO || AB$ (Construction)

$$\therefore \quad \frac{AE}{ED} = \frac{BO}{DO} \qquad \text{(By BPT)} \qquad ...(ii)$$

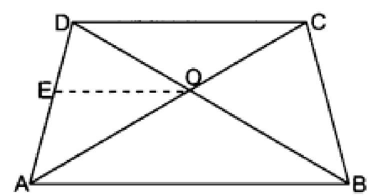


Fig. 7.28

From equations (*i*) and (*ii*)

$$\frac{AE}{ED} = \frac{AO}{CO} \quad \Rightarrow \quad EO || DC \qquad \text{(Converse of BPT)}$$

But $EO || AB$ (Construction)

AB || DC

In quad ABCD, AB || DC
⇒ ABCD is a trapezium.

(b)

Given AB||DC

$\therefore \angle ODC = \angle OBA$

(Alternate interior angles)

and
$\angle OCD = \angle OAB$

(Alternate interior angles)
$\therefore \Delta DOC \sim \Delta BOA$

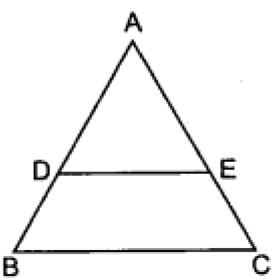
(By AA similarity criterion)

$\therefore OD/OB = OC/OA \Rightarrow x-2/x-1 = x+3/x+5$

$\Rightarrow (x-2)(x+5) = (x+3)(x-1)$

$\Rightarrow x^2+3x-10 = x^2+2x-3 \Rightarrow x=7$

Q19. (a)Given: $PS/SQ = PT/TR$ and $\angle PST = \angle PRQ$
To Prove: PQR is an isosceles triangle.
Proof: $PS/SQ = PT/TR$
By converse of BPT we get
ST || QR
$\therefore \angle PST = \angle PQR$ (Corresponding angles) ….(i)
But, $\angle PST = \angle PRQ$ (Given) ….(ii)
From equation (i) and (ii)
$\angle PQR = \angle PRQ$
$\Rightarrow PR = PQ$
So, ΔPQR is an isosceles triangle.



(b)

$\because$ BD = CE [Given]

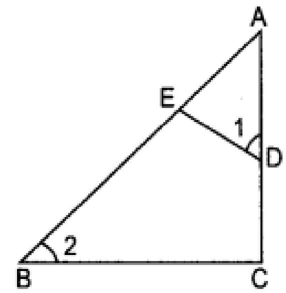
So, AD = AE $\left\{\because \frac{AD}{DB} = \frac{AE}{EC} \text{ Proved above}\right\}$

$\therefore$ AD + BD = AE + CE

$\Rightarrow$ AB = AC

$\therefore$ ΔABC is an isosceles triangle.

Q20.



Given: $\angle ADE = \angle B$, i.e. $\angle 1 = \angle 2$

To prove: $\Delta ADE \sim \Delta ABC$

Proof: In ΔADE and ΔABC

$\angle 1 = \angle 2$ [Given]

$\angle A = \angle A$ [Common]

So, $\Delta ADE \sim \Delta ABC$ [By AA similarity]

$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \quad \frac{7.6}{7.2 + 4.2} = \frac{DE}{8.4} \qquad \{\because AB = AE + BE = 7.2 + 4.2\}$$

$$\Rightarrow \quad \frac{7.6}{11.4} = \frac{DE}{8.4} \quad \Rightarrow \quad DE = \frac{7.6 \times 8.4}{11.4} = 5.6$$

Hence, $DE = 5.6$ cm.

COORDINATE GEOMETRY

DISTANCE FORMULA

The distance between any two points A(x_1, y_1) and B(x_2, y_2) is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$or \;\; AB = \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$$

Distance of a point from origin

The distance of a point P(x, y) from origin O is given by OP = $\sqrt{x^2 + y^2}$

SECTION FORMULA

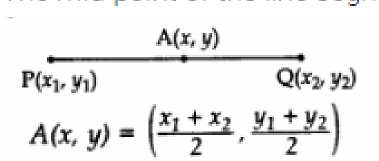
The coordinates of the point which divides the line segment joining the points A(x1, y1) and B(x2, y2) internally in the ratio $m_1 : m_2$ are:

$$\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

The ratio $m_1 : m_2$ can also be written as k : 1, The coordinates of P can also be written as P(x,y)

P(x,y) = $\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$

The mid-point of the line segment joining the points P(x1, y1) and Q(x2, y2) is



Section A (1 mark)

1 .Find the distance between the points (-6 , 7) and (-1 ,-5)

(a) 12 (b) 7 (c) 13 (d) 5

2. The mid-point of the line segment joining the points A (–2, 8) and B (– 6, – 4) is

(a) (– 4, – 6) (b) (2, 6) (c) (– 4, 2) (d) (4, 2)

3. The distance of the point P (2, 3) from the x-axis is

(a) 2 (b) 3 (c) 1 (d) 5

4. The distance between the points A (0, 6) and B (0, –2) is

(a) 6 (b) 8 (c) 4 (d) 2

5. The distance of the point P (–6, 8) from the origin is

(a) 8 (b) 2 7 (c) 10 (d) 6

Section B (2 marks)

1. Find the coordinates of the point which divides the line segment joining the points (4, -3) and (8, 5) in the ratio 3 : 1 internally

2. In what ratio does the point (-4, 6) divide the line segment joining the points A(-6, 10) and B (3, -8)?

3. If the distance between the points (2, –2) and (–1, x) is 5, the values of x is

(a) –2 (b) 2 (c) –1 (d) 1

4. The coordinates of the point P on the x axis which is equidistant from the point A (-2, 0) and B (6, 0) are ……………………………..

5. The distance between the points (m, – n) and (– m, n) is:

6. If the origin is the mid-point of the line segment joined by the points (2,3) and (x,y), then find the value of (x,y)

7. If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of its centre are (–2, 5), then find the coordinates of the other end of the diameter .

8. If the distance between the points (4,k) and (1,0) is 5 then what are the possible values of k?

9. Find the point on the X-axis which is equidistant from (2,-5) and (-2, 9).

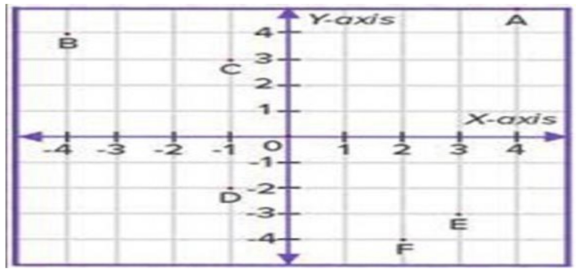
10. Find the ratio in which P (4, 5) divides line which join A (2,3) and B(7,8).

Section C (4 marks)

CASE STUDY 1

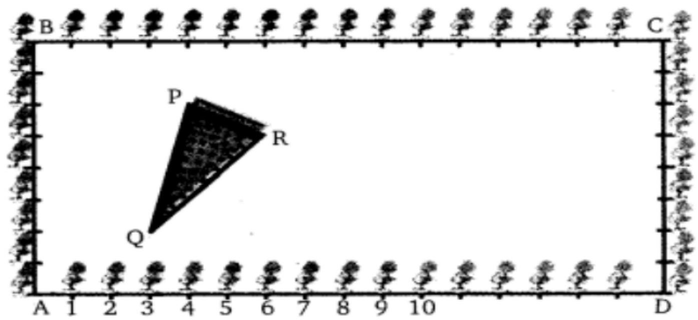
The Cartesian coordinate system is considered Inside the area of a garden to understand the heights, distances and various other mathematical measurement parameters .

The above positions A, B, C ,D ,E, F are for the plants grown in the garden . Garderner’s position is at origin (0 , 0) . He grows the plant A first , then B , C, D, E and F as according to the alphabetical manner.



I. Locate the point E.
II. Find the coordinates of the point which divides the line segment of plant C and D in 1 : 1.
III. How far apart are plants A and B grown?
IV. In what ratio line segment BE is divided at point O.

CASE STUDY 2



The class X students of a school in Krishnanagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1m from each other. There is a triangular grassy lawn in the plot as shown in the figure. The students are to saw seeds of flowering plants on the remaining area of the plot

Based on the above information, answer the following questions.

(a) Taking A as origin, find the coordinate of P.

(b) What will be the coordinate of R if C is the origin?

(c) Find the distance between P and R with respect to A as origin.

OR

Compare the distance between PQ and RQ with respect to A as origin.

Answers

Section A

1.(c)13, 2. (c) (– 4, 2), **3. (b)** 3 **4. (b)**8, **5.(c)**10

Section B (2 marks)

1.

We are given,

$(x_1, y_1) = (4, -3)$ & $1(x_2, y_2) = (8, 5)$

Let (x,y)coordinates which divides the line joining the point (x_1, y_1)

and (x_2, y_2) in ratio $m : n = 3 : 1$ internally.

$$\text{So,}(x, y) = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

$$= \left(\frac{3(8) + 1(4)}{3 + 1}, \frac{3(5) + 1(-3)}{3 + 1}\right)$$

$$= \left(\frac{28}{4}, \frac{12}{4}\right)$$

$(x, y) = (7, 3)$

2.

Solution : Let (– 4, 6) divide AB internally in the ratio k : 1.

Using the section formula, $x = \frac{m_2x_1 + m_1x_2}{m_1 + m_2}, y = \frac{m_2y_1 + m_1y_2}{m_1 + m_2}$ we get

$$y = \frac{k(-8) + 1(10)}{k + 1} = 6$$

$$\Rightarrow -8k + 10 = 6k + 6 \Rightarrow -8k - 6k = 6 - 10$$

$$\Rightarrow -14k = -4 \Rightarrow k = \frac{4}{14} = \frac{2}{7}$$

Therefore, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7.

3.

The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Therefore, the distance between the points (2, -2) and (-1, x)

$= \sqrt{(-1-2)^2 + (x-(-2))^2}$

$\Rightarrow 5 = \sqrt{(-3)^2 + (x+2)^2}$

Squaring both sides, we get

$\Rightarrow (5)^2 = \left(\sqrt{9 + (x+2)^2}\right)^2$

$\Rightarrow 25 = 9 + (x+2)^2$

$\Rightarrow 25 = 9 + x^2 + 4 + 4x$

$\Rightarrow x^2 + 4x + 13 - 25 = 0$

$\Rightarrow x^2 + 4x - 12 = 0$

$\Rightarrow x^2 + 6x - 2x - 12 = 0$

$\Rightarrow x(x+6) - 2(x+6) = 0$

$\Rightarrow (x-2)(x+6) = 0$

$\Rightarrow x = 2, \; -6$

Hence, the values of x are 2 and -6.

4. P is the midpoint of A & B.

   Coordinates of P = $(\frac{x1+x2}{2}, \frac{y1+y2}{2})$

   = $(\frac{-1+6}{2}, \frac{0+0}{2})$ = (2,0)

5. $2\sqrt{m^2 + n^2}$

6. (0, 0) is the midpoint of (2,3) and (x,y)

   = $(\frac{x1+x2}{2}, \frac{y1+y2}{2})$ = (0,0)

   = $(\frac{2+x}{2}, \frac{3+y}{2})$ = (0,0)

   = $\frac{2+x}{2}$ = 0, $\frac{3+y}{2}$ = 0

   x= -2, y= -3

7. Centre is the midpoint of the diameter

   Coordinates are (-6, 7)

8. k = +4, -4

9. (-7, 0)

10. 2 : 3

Section C

Case study - I

I. (3,-3)

II C (-1, 3), D (-1, -2)

Coordinates of the midpoint are (-1, ½)

III 8

IV B (-4, 4), E(3, -3)

Ratio is 4:3

Case Study -2

(a) (4,6)
(b) (-10,-3)
(c) $\sqrt{5}$ units

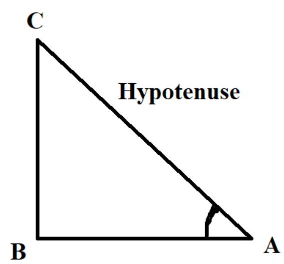
OR

PQ =$\sqrt{17}$ units, RQ= $\sqrt{18}$ units

INTRODUCTION TO TRIGONOMETRY

Trigonometric Ratios

For the right ΔABC, right-angled at ∠B, the trigonometric ratios of the ∠A are as follows:



- sin A=opposite side/hypotenuse=BC/AC
- cos A=adjacent side/hypotenuse=AB/AC
- tan A=opposite side/adjacent side=BC/AB
- cosec A=hypotenuse/opposite side=AC/BC
- sec A=hypotenuse/adjacent side=AC/AB
- cot A=adjacent side/opposite side=AB/BC

Relation between Trigonometric Ratios

- cosec θ =1/sin θ
- sec θ = 1/cos θ
- tan θ = sin θ/cos θ
- cot θ = cos θ/sin θ=1/tan θ

- Standard values of Trigonometric ratios

∠A	0^o	30^o	45^o	60^o	90^o
sin A	0	½	1/√2	√3/2	1
cos A	1	√3/2	1/√2	1/2	0
tan A	0	1/√3	1	√3	not defined
cosec A	not defined	2	√2	2/√3	1
sec A	1	2/√3	√2	2	not defined

cot A	not defined	√3	1	1/√3	0

Trigonometric Identities

The three most important trigonometric identities are:

- $\sin^2\theta + \cos^2\theta = 1$
- $1+\cot^2\theta=\text{cosec}^2\theta$
- $1+\tan^2\theta=\sec^2\theta$

One Mark

1. If cosec θ = $\frac{13}{12}$, find the value of cot θ + tan θ.
2. If tan θ = √3 , find the value of sin θ.cos θ .
3. Find the value of sin 60° cos 30° - cos 60° sin 30°.
4. Evaluate cos 1° cos 2° cos 3° …………. Cos 89° cos 90°.
5. Find the value of x , when it is given that sin (2x + 10)° = 1

Two Marks

6. If Cos (A-B) = $\frac{\sqrt{3}}{2}$ and sin (A+B) = $\frac{\sqrt{3}}{2}$, find A and B , where (A+B) and (A-B) are acute angles.
7. Evaluate : $\frac{60°+30°-245°}{30°+45°}$.
8. Prove that : $\cos^4A - \cos^2A = \sin^4A - \sin^2A$.
9. Simplify : (1-sin A)(tan A + Sec A).
10. If sin θ = $\frac{21}{29}$, evaluate $\frac{\sec\theta}{\tan\theta-\sin\theta}$.

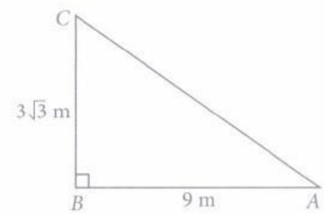
Three Marks

11. If sin 2x = sin 30°. Cos 60° + sin 60° cos 30°, then find the value of x.
12. If sin (A+B) = $\frac{\sqrt{3}}{2}$ and cos (A-B) = $\frac{\sqrt{3}}{2}$ where A and B are acute angles and A>B, then find A and B. Hence find the value of tan (A+B) + tan (A-B)
13. Prove that : $\frac{\cos\cos\theta}{1-\sin\theta} + \frac{\cos\cos\theta}{1+\sin\theta} = 2\sec\theta$
14. If $\tan\theta + \frac{1}{\tan\theta} = \sqrt{2}$, find the value of $\tan^2\theta + \frac{1}{\theta}$.
15. If $7\sin^2\theta + 3\cos^2\theta = 4$, then find $\sec\theta + \text{cosec}\theta$.

Four Marks

16. Three friends – Anshu, Vijay and Vishal are playing hide and seek in a park. Anshu and Vijay hide in the shrubs and Vishal have to

find both of them. If the positions of three friends are at A, Band C respectively as shown in the figure and forms a right angled triangle such that AB = 9 m, BC = $33\sqrt{3}$ m and ∠∠B = 90°, then answer the following questions.



a. The measure of ∠A is ……………….
b. Find the length of AC?
c. Find the values of Sin A and Cos A.

17. Prove that $tan^2A + cot^2A + 2 = sec^2A cosec^2A$.
18. If 4 sin θ = 3 cosθ, find the value of $\frac{12 sinsin\,\theta - 7 coscos\,\theta}{8 sinsin\,\theta + 3 coscos\,\theta}$.
19. The string of a kite is 45 m long. It makes an angle θ with the level ground such that sinθ= $\frac{1}{3}$. Assuming the string to be stretched and straight, find:
    a. The height of the kite above the ground level.
    b. The distance between the lower end of the string and the foot of the perpendicular from the kite to the ground.
20. Determine the value of x such that 2 $cosec^2 30°$ + x $sin^2 60°$ - $\frac{3}{4}$ $tan^2 30°$=10

<u>Five marks questions</u>

21. If tan $\theta = \frac{4}{3}$, find the value of $\sqrt{\frac{1-sin\theta}{1+sin\theta}} + \sqrt{\frac{1+sin\theta}{1-sin\theta}}$.
22. Prove that $\frac{cotA+cosecA-1}{cotA-cosecA+1} = \frac{1+cosA}{sinA}$.
23. If secA – tanA = x , show that secA+tanA=1/x and hence find the value of cosA and SinA.
24. Prove that (1+ cotA – cosecA)(1 + tanA + secA)= 2
25. Prove that : (tanθ + secθ-1)(tanθ+1+secθ)= $\frac{2sin\theta}{1-sin\theta}$.

Answer Key

1. $\frac{169}{60}$

2. $\frac{\sqrt{3}}{4}$
3. $\frac{1}{2}$
4. 0
5. 2x + 10 = 90 ⇒ x = 40°
6. A- B=30° and A+B= 60° ⇒ A = 45° and B = 15°
7. $\frac{10}{3}$
8. $Cos^4A-cos^2A = cos^2A(cos^2A - 1) = (1-sin^2A)(1-sin^2A-1) = (1-sin^2A)(-sin^2A)$
   $= sin^4A-sin^2A$
9. $(1\text{-sin }A)\left(\frac{sinsin\ A}{coscos\ A} + \frac{1}{coscos\ A}\right) = \frac{(1-sinsin\ A)(1+sinsin\ A)}{coscos\ A}$ = Cos A
10. $\frac{33}{611}$
11. 45°
12. A=45° , B= 15° and tan (A+B) + tan (A-B)=$\frac{4\sqrt{3}}{3}$
13. $\frac{cos\theta(1+sin\theta)+cos\theta(1-sin\theta)}{1-\theta} = \frac{2\ cos\theta}{\theta} = 2\ sec\theta$
14. $(tan\theta + \frac{1}{tan\theta})^2 = 2 \Rightarrow tan^2\theta + \frac{1}{\theta} = 0$
15. From given get $\theta = 30° \Rightarrow sec\theta + cosec\theta = 2 + \frac{2}{\sqrt{3}}$
16. A. 30° B. $6\sqrt{3}$ C. SinA=1/2 , CosA=$\frac{\sqrt{3}}{2}$
17.

$$LHS = \tan^2 A + \cot^2 A$$

$$= \frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A}$$

$$= \frac{\sin^4 A + \cos^4 A}{\sin^2 A \cos^2 A}$$

$$= \frac{(\sin^2 A + \cos^2 A)^2 - 2\sin^2 A \cos^2 A}{\sin^2 A \cos^2 A}$$

$$= \frac{1 - 2\sin^2 A \cos^2 A}{\sin^2 A \cos^2 A}$$

$$= \frac{1}{\cos^2 A}.\frac{1}{\sin^2 A} - 2$$

$$= \sec^2 A.\text{cosec}^2 A - 2$$

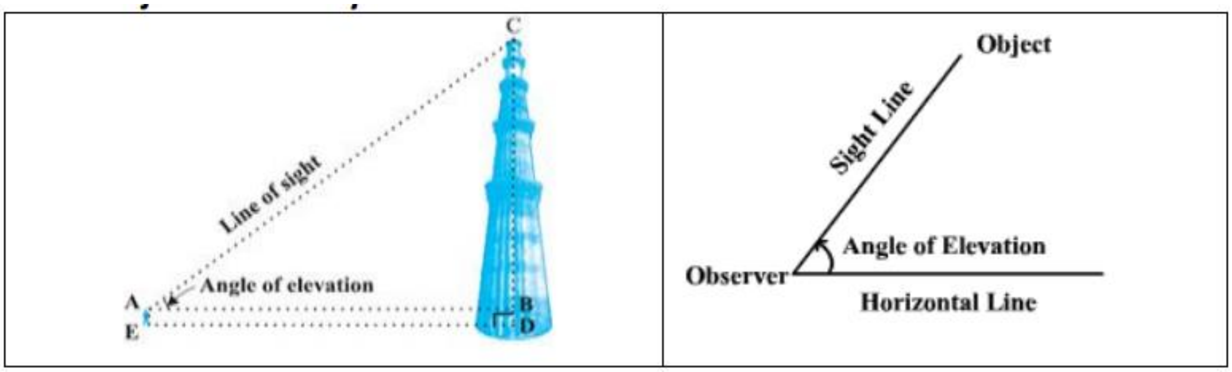
18. $\frac{2}{9}$

19. A. height of kite above ground level = 15 m B. $30\sqrt{2}$ m
20. X = 3
21. $\frac{10}{3}$
22. LHS= $\frac{(cotA+cosecA)-(A-A)}{cotA-cosecA+1}$ = $\frac{(cotA+cosecA)(1-cosecA+cotA)}{cotA-cosecA+1}$ = cotA+cosecA =
= $\frac{cosA+1}{sinA}$ = RHS
23. cosA= $\frac{2x}{x^2+1}$ SinA= $\frac{x^2-1}{x^2+1}$
24. $(1+\frac{cosA}{sinA} - \frac{1}{sinA})(1+\frac{sinA}{cosA} + \frac{1}{cosA}) = (\frac{sinA+cosA-1}{sinA})(\frac{cosA+sinA+1}{cosA}) = \frac{(sinA+cosA)^2-1}{sinAcosA}$
= $\frac{2sinAcosA}{sinAcosA}$ = 2
25. $(\frac{sin\theta}{cos\theta} + \frac{1}{cos\theta} -1)(\frac{sin\theta}{cos\theta} +1 + \frac{1}{cos\theta}) = \frac{(sin\theta+1-cos\theta)(sin\theta+1+cos\theta)}{\theta} = \frac{2sin\theta}{1-sin\theta}$

APPLICATIONS OF TRIGONOMETRY

ANGLE OF ELEVATION

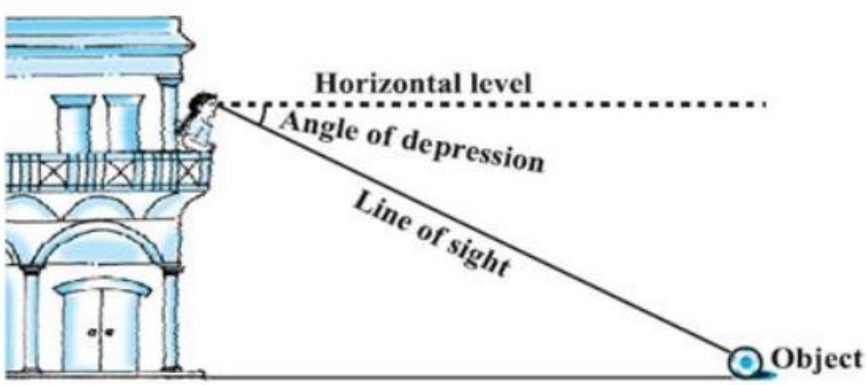
In the below figure, the line AC drawn from the eye of the student to the top of the minar is called the line of sight. The student is looking at the top of the minar. The angle BAC, so formed by the line of sight with the horizontal, is called the angle of elevation of the top of the minar from the eye of the student. Thus, the line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer

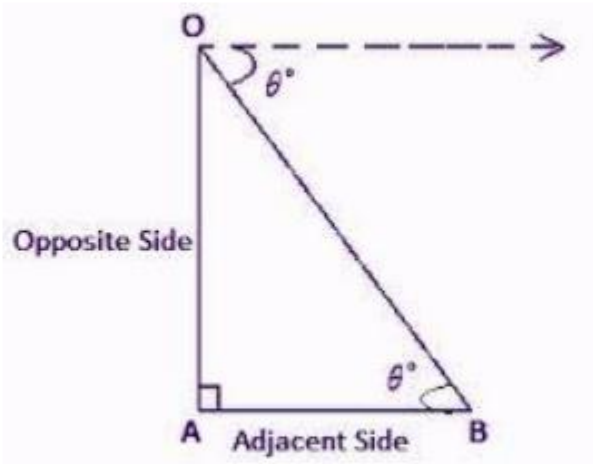


The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object

ANGLE OF DEPRESSION

In the below figure, the girl sitting on the balcony is looking down at a flower pot placed on a stair of the temple. In this case, the line of sight is below the horizontal level. The angle formed by the line of sight with the horizontal is called the angle of depression.





Thus, the angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed

1 MARK QUESTIONS

1. The line drawn from the eye of an observer to the point in the object viewed by the observer, is known as?

   A. Line of sight   B. Angle of elevation   C. Angle of depression   D. None of these

2. The angle formed by the line of sight with the horizontal when the point on the object which is being viewed is above the horizontal level, is known as?

   A. Line of sight   B. Angle of elevation   C. Angle of depression   D. None of these

3. The angle formed by the line of sight with the horizontal when the point on the object which is being viewed is below the horizontal level, is known as?

   A. Line of sight   B. Angle of elevation   C. Angle of depression   D. None of these

4. A tower stands vertically on the ground. From a point on the ground , which is 15m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Then height of the tower is

A. 15m   B.$15\sqrt{3}$ m   C. $\frac{15}{\sqrt{3}}$ m   D. None of these

5. A pole 6m high casts shadow $2\sqrt{3}$ m long on the ground , then the sun's elevation is

   A. 60° B. 30° C. 45° D.90°

2 MARKS QUESTIONS

6. The angle of elevation of the top of a tower from a point on the ground, which is 20m away from the foot of the tower, is 60^0 . Find the height of the tower.:

   A. 10√3 m B. 30√3 m C. 20√3 m D. None of these.

7. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.

   A. 10 m B. 19 m C. 20 m D. None of these

8. If the ratio of the height of a tower and the length of its shadow is √3:1, what is the angle of elevation of the Sun?

   A.30° B. 60° C. 45° D. None of these

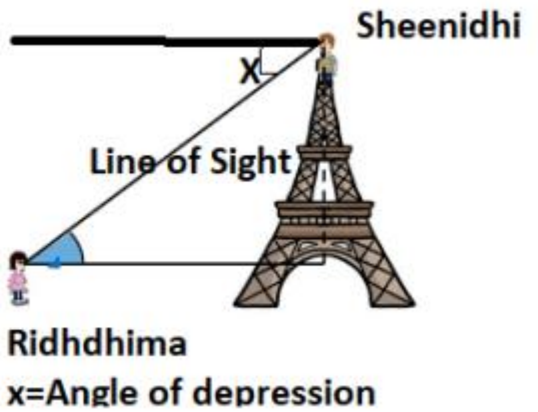
9. If the angle of elevation of a tower from a distance of 100m from its foot is 60° , then the height of the tower is

   A. 100√3 m B. 200/√3 m C. 50√3 m D. 100/√3 m

10. A tower is 50m high, its shadow ix 'x' metres shorter when the sun's altitude is 450 than when it is 300 . Find the value of 'x'

    A. 100√3 m B. 200/√3 m C. 50√3 m D. 50(√3 – 1) m

4 MARKS QUESTIONS - Case Study Questions



11.

Ridhdhima is driving a scooter. On her way , she approaches a tower and observes that Shreenidhi is standing on the top of the tower. A sign board beside the tower read – angle of depression =60°. The distance from the tower at which Ridhdhima stops her scooter is 50 m.

I. Is the angle of elevation from Ridhdhima's scooter to the top of tower, where Shreenidhi is standing the same as the angle of depression? Yes/No

II. What is the angle of elevation from Ridhdhima's scooter to the top of the tower where Sheenidhi is Standing?

a. 60° b. 30° c. 45° d.90°

III. What is the height of the tower?

a. 50 b. $50\sqrt{3}$ m c. 60 d. $60\sqrt{3}$ m

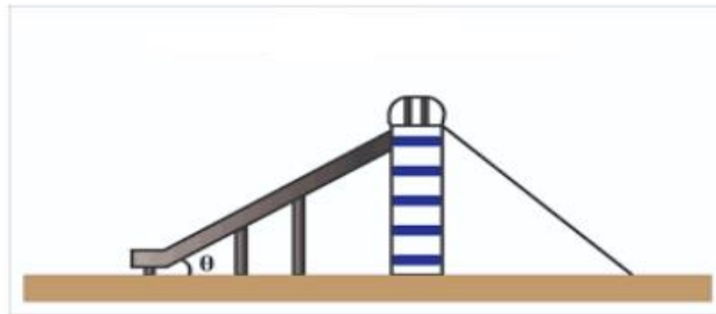
IV. What is the length of the line of sight?

a. 50m b. 70m c. 100m d. 120m

V. Will the angle of elevation increase or decrease as the scooter approaches the tower?

a. Increases b. decreases

12.



Raj went to a Mayur Bag. He went up the slide to play. The angle of elevation θ of the slide is 30°. But the base from which the angle of elevation is measured is 5 m above the ground level and the distance from the staircase is 10m. ($\sqrt{3}$ = 1.732)

I. What is the distance of the staircase from the point from which the angle of elevation of the slide is measured?

a. 5m b. 10m c. 15m d. 20m

II. What is the angle of depression from the top ofb the slide to its base?

a. 60° b. 30° c. 45° d.90°

III. What is the height of the staircase?

a. 15.77m b. 10.77m c. 5.77m d. None of these

IV. What is the length of the slide?

a. 9.874 m b. 8.46m c. 11.547m d. None of these

V. Will the angle of elevation increase or decrease if the staircase was made taller?

a. Increases b. Decreases

13. 

A group of students of class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.

I. What is the angle of elevation if they are standing at a distance of 42m away from the monument?

a. 60° b. 30° c. 45° d.0°

II. They want to see the tower at an angle of 60o . So, they want to know the distance where they should stand and hence find the distance

a. 25.24 m b.20.12 m c. 42 m d. 24.64 m

III. If the altitude of the Sun is at 60o , then the height of the vertical tower that will cast a shadow of length 20 m is

a. $20\sqrt{3}$ m b. $20/\sqrt{3}$ m c. $15/\sqrt{3}$ m d. $15\sqrt{3}$ m

IV. The ratio of the length of a rod and its shadow is 1:1. The angle of elevation of the Sun is

a. 30° b. 45° c. 60° d.90°

V. The angle formed by the line of sight with the horizontal when the object viewed is below the horizontal level is

a. Corresponding angle b. Angle of elevation

c. Angle of depression d. Complete angle

14. A vertically straight tree, 15 m high, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break?

15. A person observed the angle of elevation of a tower as 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower.

ANSWER KEY

1. (A)
2. (B)
3. (C)
4. (B)
5. (A)
6. (C)
7. (B)
8. (A)
9. (D)
10. (D)
11. i. A	ii. A	iii. B	iv. C	v. A
12. i. B	ii. B	iii. C	iv. C	v. A
13. i. C	ii. A	iii. A	iv. B	v. A

CIRCLES

Circle: A circle is a collection of all points in a plane which are at a constant distance from a fixed point.

Centre: The fixed point is called the centre.

Radius: The constant distance from the centre is called the radius.

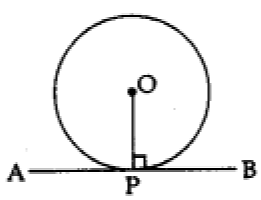
Chord: A line segment joining any two points on a circle is called a chord.

Diameter: A chord passing through the centre of the circle is called diameter. It is the longest chord.

Tangent: When a line meets the circle at one point or two coincidings The line is known as points, a tangent.

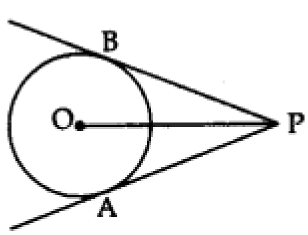
The tangent to a circle is perpendicular to the radius through the point of contact.

$\Rightarrow OP \perp AB$



The lengths of the two tangents from an external point to a circle are equal.

$\Rightarrow AP = PB$



1 MARK QUESTIONS

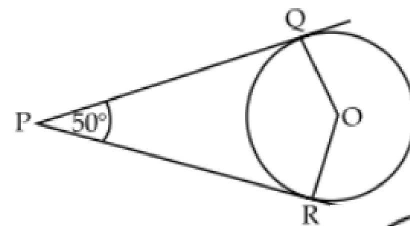
1. A line which is perpendicular to the radius of the circle through the point of contact is:

   A. Tangent B. Chord C. Segment D. Normal

2. Number of tangents to a circle which are parallel to a secant is:

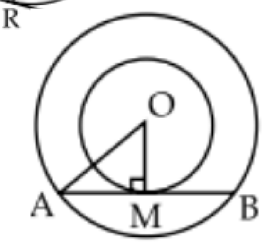
   A. 1 B. 2 C. 3 D. Infinite

3. In the given quadrilateral, OQPR, <QOR is equal to:



   A. 120° B. 130° C. 145° D. 110°

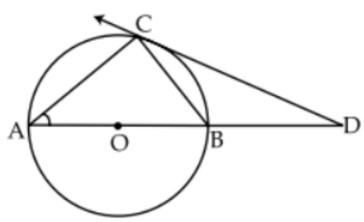
4. In fig. if OA = 5cm, OM = 3cm, the length of chord AB (in cm) is:



   A. 8 B. 10 C. 6 D. 4

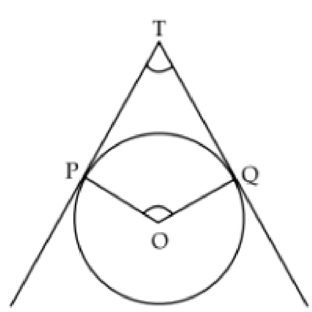
5. In the fig. AB is a diameter and AC is a chord of a circle such that $< BAC = 30^o$. If DC is a tangent, then ΔBCD is:

   A. Equilateral B. Isosceles C. Right angled D. Acute angled



2 MARKS QUESTIONS

6. In Fig. if from an external point T, TP and TQ are two tangents to a circle with centre O so that $< POQ = 110°$, then $< PTQ$ is:

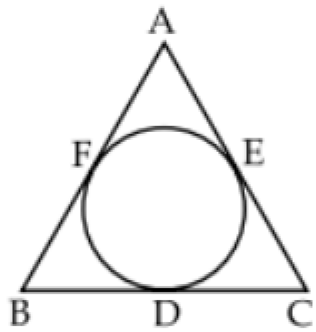


A.60° B. 70° C. 80° D. 90°

7. From a point P which is at a distance of 13cm from the centre O of a circle of radius 5cm, the pair of tangents PQ and PR to the circle are drawn. What are the lengths (in cm) of tangents PQ and PR?

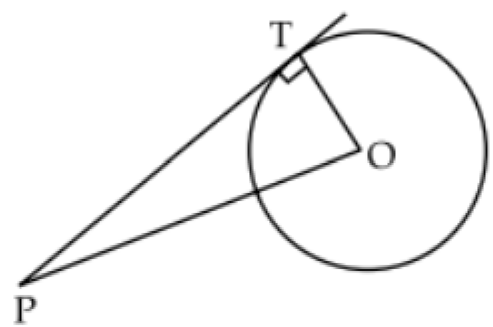
A. 13,12 B. 13, 13 C. 12,12 D. 12,18

8. In the fig. if the semi perimeter of ΔABC = 23cm, then AF + BD + CE is:



A. 46cm B. 11.5cm C. 23cm D. 34.5cm

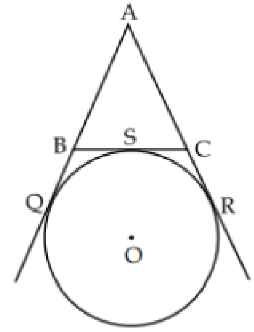
9. In the fig. PT is a tangent to a circle with centre O. If PT = 30 cm and diameter of circle is 32cm, then the length of the line segment OP will be:



A. 68cm B. 34cm C. 17cm D. 34.8cm

10. In fig. AQ, AR and BC are tangents to a circle with centre O, If AB = 7cm, BC = 5cm AC = 5cm, then the length of tangent AQ is:

A. 5cm B. 7cm C. 8.5cm D. 17cm



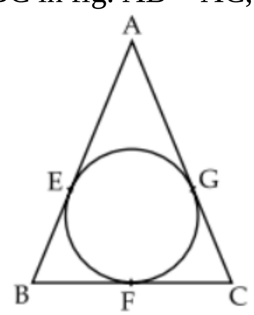
3 MARKS QUESTIONS

11. Prove that the tangents drawn at the end- points of the diameter of a circle are parallel.

12. Two concentric circles have centre O, OP= 4cm, OB = 5cm. AB is a chord of the outer circle and tangent to the inner circle at P. Find the length of AB

13. Two tangents PA and PB are drawn to a circle with centre O such that $< APB = 120°$. Prove that OP=2AP

14. In the isosceles triangle ABC in fig. AB = AC, show that BF = FC



15. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is _______

5 MARKS QUESTIONS

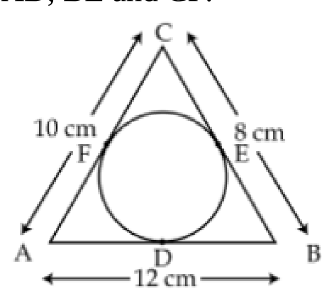
16. Prove that a tangent at any point of a circle is perpendicular to the radius through the point of contact.

17. Prove that the lengths of tangents drawn from an external point to a circle are equal.

18. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB+CD = AD + BC.

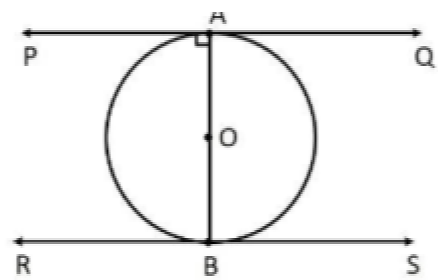
19. Prove that a parallelogram circumscribing a circle is a rhombus.

20. In the fig. a circle is inscribed in a ΔABC with sides AB = 12cm, BC = 8 cm and AC = 10cm. Find the lengths of AD, BE and CF.



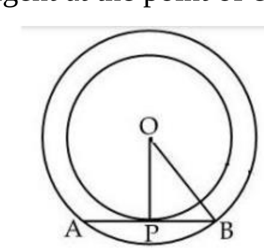
ANSWER KEY

1. (A)
2. (B)
3. (B)
4. (A)
5. (B)
6. (B)
7. (C)
8. (C)
9. (B)
10. (C)
11.



Let AB be a diameter of the circle.
Two tangents PQ and RS are drawn at points A and B respectively
Radius drawn to these tangents will be perpendicular to the tangents.
Thus, $OA \perp PQ$ and $OB \perp RS$
$\angle OAP = 90^\circ$
$\angle OAQ = 90^\circ$
$\angle OBR = 90^\circ$
$\angle OBS = 90^\circ$.
It can be observed that $\angle OAP = \angle OBS$ (Alternate interior angles)
$\angle OAQ = \angle OBR$ (Alternate interior angles)
Since alternate interior angles are equal, lines PQ and RS will be parallel

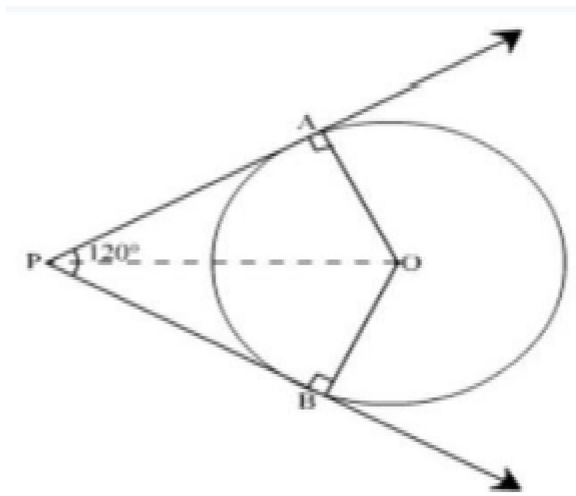
12. We know that the radius is perpendicular to the tangent at the point of contact
Therefore, $\angle OPB = 90^\circ$
In right triangle OPB,
$OB^2 = OP^2 + PB^2$
$(5)^2 = (4)^2 + PB^2$
$PB^2 = 25 - 16 = 9$



OP = 4 cm, OB = 5 cm

PB = 3 cm
We know that perpendicular from the centre to the chord bisect the chord.
Therefore, AB = 2PB = 6 cm

13.



In ΔOAP and ΔOBP,
OP = OP (Common)
∠OAP = ∠OBP (90°) (Radius is perpendicular to the tangent at the point of contact)
OA = OB (Radius of the circle) ∴ ΔOAP is congruent to ΔOBP (RHS criterion)
∠OPA = ∠OPB = 120°/2 = 60° (CPCT)
In ΔOAP,
Cos ∠OPA = cos 60° = AP/OP
Therefore, 1/2 =AP/OP. Thus, OP = 2AP. Hence, proved.

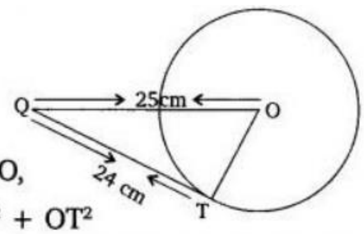
14. AB= AC (given), ie AE +BE = AG + GC
BE = GC (Length of tangents drawn from an external point to a circle are equal)
BF = CF (∵ BE = BF and GC = CF)

15.

Justification:

Let OT be x cm.

Then in right ΔQTO,



$$QO^2 = QT^2 + OT^2$$

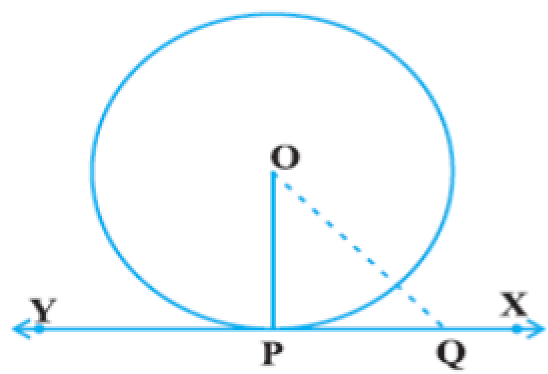
[By Pythagoras' Theorem]

$\Rightarrow \quad (25)^2 = (24)^2 + x^2$

$\Rightarrow \quad x^2 = 625 - 576 = 49$

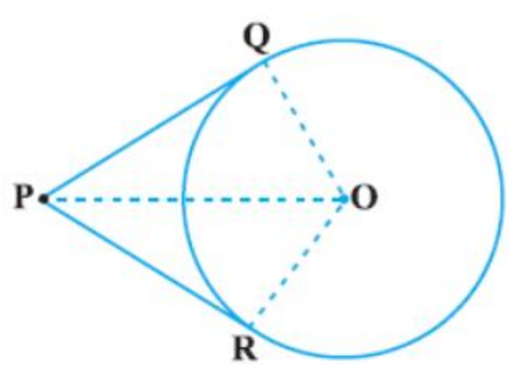
$\Rightarrow \quad x = \sqrt{49} = \mathbf{7\ cm.}$

16.



Given: A circle with centre O and a tangent XY to the circle at a point P. 59 | P a g e
Required to prove : OP is perpendicular to XY.
Constructions: Take a point Q on XY other than P and join OQ
Proof: According to the diagram The point Q must lie outside the circle. if Q lies inside the circle, XY will become a secant and not a tangent to the circle). Therefore, OQ is longer than the radius OP of the circle. That is, OQ > OP. Since this happens for every point on the line XY except the point P, OP is the shortest of all the distances from the point O to the points of XY. So OP is perpendicular to XY.

17.



Given: A circle with centre O, a point P lying outside the circle and two tangents PQ, PR on the circle from P

Required to prove : PQ = PR.

Constructions: join OP, OQ and OR.

Proof: According to the diagram OQP and ORP are right angles, Because these are angles between the radii and tangents, and according to Theorem 10.1 they are right angles. Now in right triangles OQP and ORP,

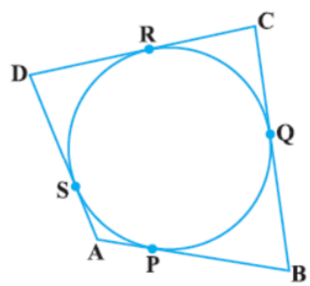
OQ = OR (Radii of the same circle)

OP = OP (Common)

Δ OQP $\cong$ Δ ORP (RHS)

PQ = PR (CPCT)

18.



Given: ABCD is a quadrilateral which circumscribing a circle with center O P,Q,R,S are the points of contact of sides AB,BC,CD,DA respectively

Required to Prove : AB + CD = AD + BC

Proof: According to diagram

AP = AS ...(i)

[Lengths of tangents from an external point are equal]

BP = BQ ...(ii)

CR = CQ ...(iii)

DR = DS ...(iv)

Adding equations (i), (ii), (iii) and (iv), we get

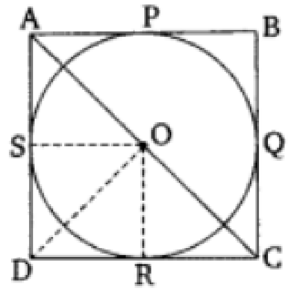
AP + BP + CR + DR = AS + BQ + CQ + DS

⇒ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

⇒ AB + CD = AD + BC

Hence proved.

19.



Given: ABCD is a parallelogram which circumscribing a circle with center O P,Q,R,S are the points of contact of sides AB,BC,CD,DA respectively

Required to Prove: ABCD is a rhombus

Proof: According to diagram

AP = AS ...(i)

[Lengths of tangents from an external point are equal]

BP = BQ ...(ii)

CR = CQ ...(iii)

DR = DS ...(iv)

Adding equations (i), (ii), (iii) and (iv), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

⇒ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

⇒ AB + CD = AD + BC

But AB = CD AND AD = BC (given) So 2AB = 2 AD AB = AD Also AB = AD = BC = CD. Hence ABCD is a rhombus.

20. Let AD= x cm

BD = 12 – x

BE = 12 – x

CE = 8 – (12 – x)

CE = x – 4 …….. (i)

AF = x

CF = 10 – x --------(ii)

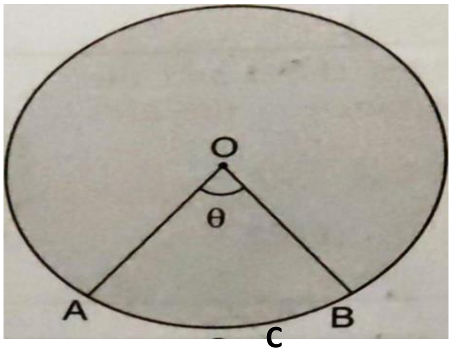
From (i) and (ii) , we get x- 4 = 10-x

x = 7 cm. Therefore, AD = 7 cm BE = 5cm CF = 3cm

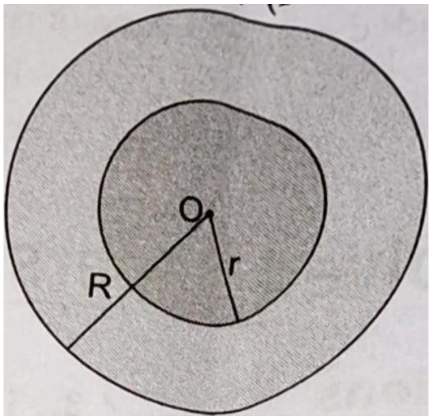
AREAS RELATED TO CIRCLES

MAIN CONCEPTS

1. Diameter is the longest chord of a circle.
    - Diameter = 2 x radius.
    - Central angle of arc AB = angle AOB = θ (say) (Refer to figure)
2. If θ is the angle at the centre of the circle, then arc AB is called minor arc and the arc BA is called major arc.



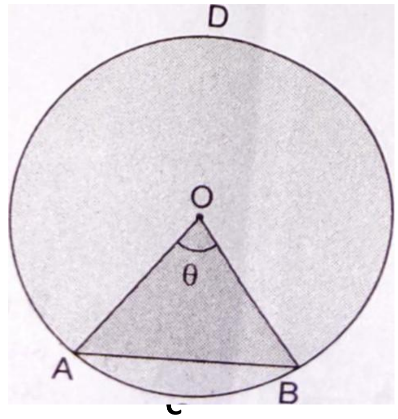
3. An arc whose length is less than the arc of a semicircle is called a minor arc. An arc whose length is more than the arc of a semicircle is called a major arc.
4. The segment containing the minor arc is called a minor segment while the segment containing the major arc is called the major segment.
5. If arc AB is a minor arc then OACBO is called the minor sector of the circle and the remaining part of the circle is called the major sector of the circle.
6. For a circle, having radius = r,
    - Area = πr^2 ;
      Circumference = 2 πr
    - Area of the semicircle = ½ πr^2
7. Perimeter of the semicircle = $\pi r + 2r$,
8. Area of a ring: Let R and r be the outer of and inner radii of a circle.
   Then area of a ring = $\pi(R^2 - r^2)$



9. Let an arc AB makes an angle θ at the centre of the circle with radius r. Then

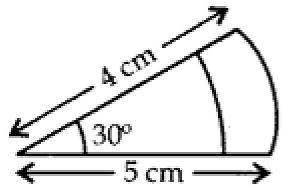
- length of arc AB = l = $2\pi r\theta/ 360°$

- Area of sector OACBO = $\pi r^2\theta / 360°$ = $1/2lr$
- Perimeter of sector OACBO = 2 + $2\pi r\theta/360°$ = $2r + l$
- Area of minor segment ACBA = area of sector OACBO - area of triangle OAB
- Area of major segment BDAB = area of circle - area of minor segment.



OBJECTIVE TYPE QUESTIONS (1 MARK EACH)

1. If the area of a circle is equal to sum of the areas of two circles of diameters 10 cm and 24 cm, calculate the diameter of the larger circle (in cm).
   Solution:
   $\pi R^2 = \pi r^2{}_1 + \pi r^2{}_2$
   $\pi R^2 = \pi(r^2{}_1 + \pi r^2{}_2)$ [r_1 = 102 = 5cm, r_2 = 242 = 12 cm]
   $R^2 = 5^2 + 12^2 = 25 + 144$
   $R^2 = 169 = 13$ cm
   ∴ Diameter = 2(13) = 26 cm
2. The circumference of a circle is 22 cm. Calculate the area of its quadrant (in cm^2).
   Solution:
   Circumference of a circle = 22 cm $2\pi r$ = 22 cm
   $$2 \times \frac{22}{7} \times r = 22 \text{ cm}$$
   $$r = \frac{22 \times 7}{22 \times 2} = \frac{7}{2} \text{ cm}$$
   $$\therefore \quad \text{Area of quadrant} = \frac{1}{4}\pi r^2$$
   $$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8} \textbf{ cm}^2$$
3. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = 22/7$, calculate the circumference (in cm) of the circle.
   Solution:
   $2\pi r - r = 37 \Rightarrow r(2\pi - 1) = 37$
   $$\Rightarrow \quad r\left(\frac{44}{7} - 1\right) = 37 \qquad \Rightarrow \qquad r\left(\frac{37}{7}\right) = 37$$
   $$\Rightarrow \quad r = 37 \times \frac{7}{37} = \mathbf{7 \text{ cm}}$$
   $$\textbf{Circumference of the circle} = 2\pi r = 2 \times \frac{22}{7} \times 7$$
   $$= \mathbf{44 \text{ cm}}$$
4. If π is taken as 22/7, calculate the distance (in metres) covered by a wheel of diameter 35 cm, in one revolution.
   Solution:
   Radius (r) = 352
   Required distance = Perimeter = $2\pi r$
   = 2 × 227 × 357 cm = 110 cm or 1.1 m
5. In the figure, find the area of shaded region.



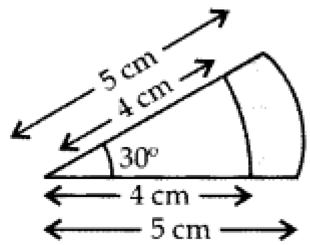
Solution:
Area of shaded region

$= \frac{30}{360}(\pi \times 5^2 - \pi \times 4^2)$

$= \frac{1}{12}\pi(25 - 16)$

$= \frac{1}{12} \times \frac{22}{7} \times 9$

$= \frac{33}{14}$ = **2.36 sq. cm**



VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.
   Solution:

Here $\theta = \frac{360^\circ}{60 \text{ m}} \times 5 \text{ m} = 30^\circ$ ...[$\because$ **1 hour = 60 minutes**

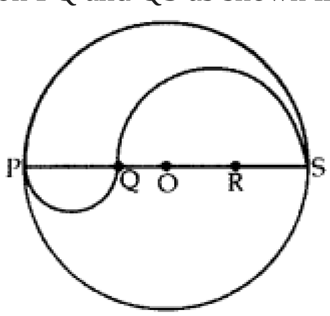
r(radius) = 14 cm

$\therefore$ **Required area** $= \frac{\theta}{360}\pi r^2$

$= \frac{30}{360} \times \frac{22}{7} \times 14 \times 14$

$= \mathbf{\frac{154}{3}}$ **cm²** or **$51.\overline{3}$ cm²**

2. PQRS is a diameter of a circle of radius 6 cm. The equal lengths PQ, QR and RS are drawn on PQ and QS as shown in Figure. Find the perimeter of the shaded region.



Solution:

Radius OS = 6 cm

∴ Diameter PS = 12 cm

∵ PQ, QR and RS, three parts of the diameter are equal.

∴ PQ = QR = RS = 4 cm

and QS = 2 × 4 = 8 cm

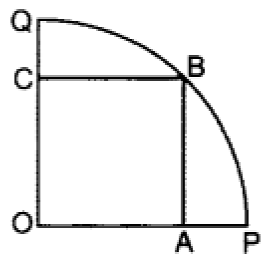
∴ Required perimeter

$= \text{arc } \widehat{PS} + \text{arc } \widehat{QS} + \text{arc } \widehat{PQ}$

$= \pi \times 6 + \pi \times 4 + \pi \times 2$

$= 6\pi + 4\pi + 2\pi = \mathbf{12\pi \text{ cm}}$

3. In Figure, a square OABC is inscribed in a quadrant OPBQ of a cirlce. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)



Solution:

Diagonal of the square (OB) = Side 2−√

∴ r = 202−√ cm ⋯[Side of square, OA = 20 cm

∴ θ = 90°

ar(Shaded region) = ar(Quad. Sector) – ar(Square)

$= \frac{\theta}{360} \pi r^2 - (\text{Side})^2$

$= \frac{1}{4} \pi r^2 - (20)^2$

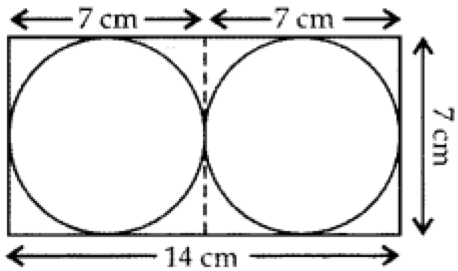
$= \frac{1}{4} \times 3.14 \times (20\sqrt{2})^2 - 400$

$= \frac{314}{400} \times 400 \times 2 - 400$

$= \quad 628 - 400 = \mathbf{228 \text{ cm}^2}$

4. Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular card board of dimensions 14 cm × 7 cm. Find the area of the remaining card board. [Use π = 22/7]

Solution:



Here r = 72 cm, L = 14 cm, B = 7 cm
Area of the remaining card board
= ar(rectangle) – 2(area of circle)
= L x B – $2\pi r^2$)
= 14 × 7 – 2 × 227 × 72 × 72
= 98 – 77 = 21 cm^2

5. Find the area of a quadrant of a circle, where the circumference of circle is 44 cm. (Use π = 22/7]
   Solution:
   Circumference of a circle = 44 cm
   ⇒ $2\pi r$ = 44 cm

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 44 \text{ cm}$$

$$\Rightarrow \quad r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

$$\text{Area of a quadrant} = \frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7$$

$$\therefore \quad \text{Area of quadrant} = \frac{77}{2} = \mathbf{38.5\ cm^2}$$

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

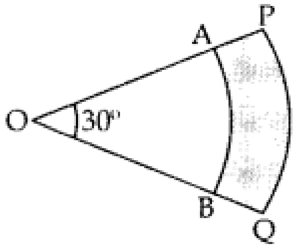
1. Area of a sector of a circle of radius 14 cm is 154 cm^2. Find the length of the corresponding arc of the sector. [Use π = 22/7]
   Solution:
   Area of sector = 154 cm^2

$$\frac{1}{2} lr = 154 \quad \Rightarrow \quad \frac{1}{2}(l)(14) = 154$$

$$\Rightarrow \quad 7l = 154 \quad \Rightarrow \quad l = 22 \text{ cm}$$

$$\therefore \quad \textbf{Length of the corresponding arc, } l = \mathbf{22\ cm}$$

2. In the Figure, PQ and AB are respectively the arcs of two concentric circles of a radii 7 cm and 3.5 cm and centre O. If ∠POQ = 30°, then find the area of the shaded region. [Use π = 22/7]



Solution:

Area of sector with radius 7 cm

$$= \pi \times 7 \times 7 \times \frac{30}{360} = \frac{49\pi}{12}$$

Area of sector with radius 3.5 cm

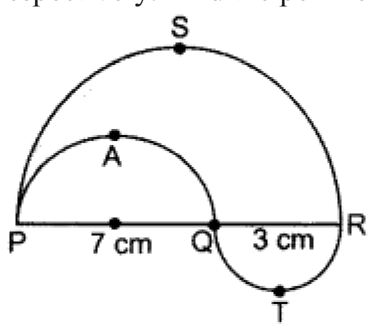
$$= \pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{30}{360} = \frac{49\pi}{48}$$

$\therefore$ **Area of the shaded region**

$$= \frac{49\pi}{12} - \frac{49\pi}{48} = \frac{196\pi - 49\pi}{48}$$

$$= \frac{147\pi}{48} = \frac{147}{48} \times \frac{22}{7} = \frac{77}{8} = \mathbf{9.625\ sq.\ cm}$$

3. In the figure, PSR, RTQ and PAQ are three semicircles of diameters 10 cm, 3 cm and 7 cm respectively. Find the perimeter of the shaded region. [Use $\pi = 3.14$]

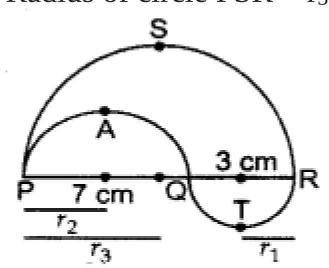


Solution:

Radius of circle QTR = r_1 = 32 = 1.5 cm

Radius of circle PAQ = r_2 = 72 = 3.5 cm

Radius of circle PSR = r_3 = 7+32 = 5 cm



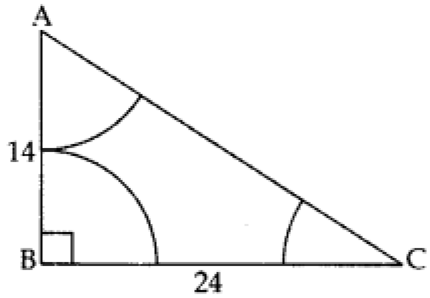
Perimeter of the shaded region

$= \pi r_1 + \pi r_2 + \pi r_3$

$= \pi(r_1 + r_2 + r_3)$

= 3.14(1.5 + 3.5 + 5)
= 3.14(10) = 31.4 cm

4. Question 15.
In Figure, ABC is a triangle right angled at B, with AB = 14 cm and BC = 24 cm. With the vertices A, B and C as centres, arcs are drawn, each of radius 7 cm. Find the area of the shaded region. (Use π = 22/7)



Solution:
Let $\angle BAC = \theta_1$, $\angle ABC = \theta_2$ and $\angle ACB = \theta_3$

Area of the shaded region
= ar(ΔABC) − [ar(sector A) + ar(sector B) + ar(sector C)]

$$= \frac{1}{2} \times AB \times BC - \left[\frac{\theta_1}{360}\pi r^2 + \frac{\theta_2}{360}\pi r^2 + \frac{\theta_3}{360}\pi r^2\right]$$

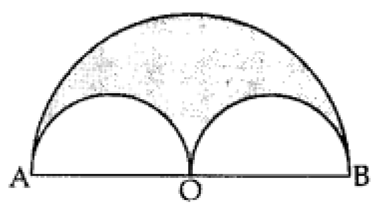
$$= \frac{1}{2} \times 14 \times 24 - \frac{\pi r^2}{360}(\theta_1 + \theta_2 + \theta_3)$$

$$= 168 - \frac{1}{360} \times \frac{22}{7} \times 7 \times 7 \times 180$$

$$\text{...}[\because \theta_1 + \theta_2 + \theta_3 = 180$$

$$= 168 - 77 = \mathbf{91\ cm^2}$$

5. In Figure, a semi-circle is drawn with O as centre and AB as diameter. Semi-circles are drawn with AO and A OB as diameters. If AB = 28 m, find the perimeter of the shaded region. [Use π = 22/7]



Solution:

$$R \text{ (Radius)} = OA = \frac{28}{2} = 14 \text{ cm}$$

$$r \text{ (radius)} = \frac{OA}{2} = \frac{14}{2} = 7 \text{ cm}$$

$\therefore$ **Perimeter of the shaded region**

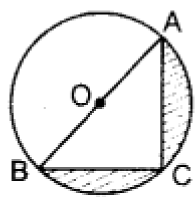
$$= \pi R + \pi r + \pi r \quad \Rightarrow \quad \pi(R + r + r)$$

$$= \frac{22}{7}(14 + 7 + 7)$$

$$= \frac{22}{7} \times 28 = \mathbf{88 \text{ cm}}$$

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

1. In Figure, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (Take $\pi = 3.14$)



Solution:

$\angle ACB = 90°$ …[Angle in a semi-circle

$\therefore AC^2 + BC^2 = AB^2$ …[Pythagoras' theorem

$(12)^2 + BC^2 = (13)^2$

$144 + BC^2 = 169$

$BC^2 = 169 - 144 = 25$

BC = + 5 cm

$\therefore$ Radius, $r = OA = OB = \frac{AB}{2} = \frac{13}{2}$ cm

Area of shaded region

$$= \text{Area (semi-circle)} - \text{Area } (\Delta ACB)$$

$$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times \text{base} \times \text{corres. altitude}$$

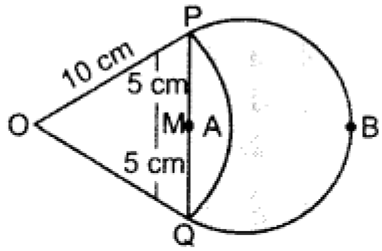
$$= \frac{1}{2} \times 3.14 \times \left(\frac{13}{2}\right)^2 - \frac{1}{2} \times 5 \times 12$$

$$= \frac{3.14 \times 169}{8} - 30$$

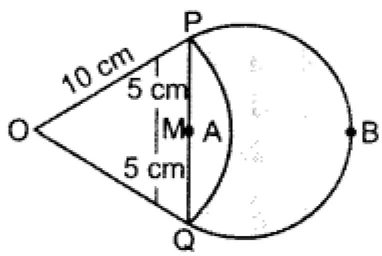
$$= 66.33 - 30$$

$$= \mathbf{36.33 \text{ cm}^2}$$

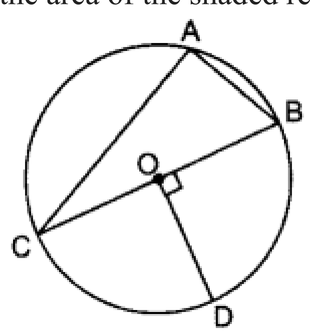
2. In Figure, are shown two arcs PAQ and 0 PBQuestion Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi–circle drawn on PQ as diameter with centre M. If OP = PQ = 10 cm, show that area of shaded region is 25(3−√=π6)cm^2.



Solution:
OP = OQ = 10 cm (Tangents drawn from an external point are equal
PQ = 5 + 5 = 10 cm
OP = OQ = PQ = 10 cm … [sides are equal
∴ ΔPOQ is an equilatral Δ.
⇒ ∠POQ = 60° ⋯[Angles of an equilateral Δ
Side = 70 cm, r = 5 cm, θ = 60°, R = 10 cm
Area of the shaded region = Area of ΔOPQ + Area of Semi–circle(PBQM) − Area of sector (OPAQ)



3. In Figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and ∠BOD = 90°. Find the area of the shaded region. (Use π = 3.14)



Solution:
∠BAC = 90° ⋯[Angle in a semi–circle
Int rt. ΔBAC,
$BC^2 = AC^2 + AB^2$ ⋯(Pythagoras' theorem
= 242 + 72

$= 576 + 49 = 625$

$BC = \sqrt{625} = 25$ cm

$\therefore$ Radius, $r = OB = \frac{BC}{2} = \frac{25}{2}$ cm

Area of shaded region = ar(circle) − [ar(ΔBAC) + ar(quadrant)]

$= \pi r^2 - \left[\frac{1}{2} \times AC \times AB + \frac{1}{4}\pi r^2\right]$

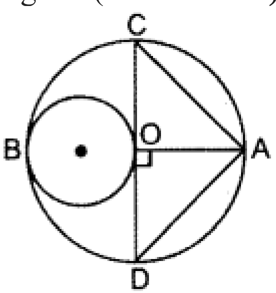
$= 3.14 \times \frac{25}{2} \times \frac{25}{2} - \left[\frac{1}{2} \times 24 \times 7 + \frac{1}{4} \times 3.14 \times \frac{25}{2} \times \frac{25}{2}\right]$

$= 490.625 - [84 + 122.656]$

$= 490.625 - 206.656$

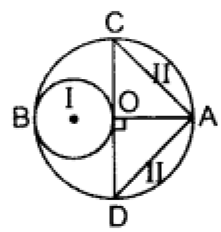
$= 283.969 =$ **284 sq. cm**

4. In Fig., AB and CD are two diameters of a circle with centre 0, which are perpendicular to each other. OB is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. (Use π = 22/7)



Solution:

Here, r = 7/2 cm, R = 7 cm



Area (I) $= \pi r^2$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{2} = 38.5 \text{ cm}^2$

Area (II) = ar(semi-circle) − ar(ΔDAC)

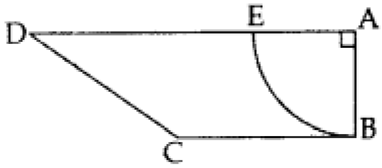
$= \frac{1}{2}\pi R^2 - \left(\frac{1}{2} \times CD \times OA\right)$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 - \left(\frac{1}{2} \times 14 \times 7\right)$

$= 77 - 49 = 28 \text{ cm}^2$

$\therefore$ **Required shaded area** = 38.5 + 28 = **66.5 cm²**

5. In Figure, ABCD is a trapezium of area 24.5 sq. cm. In it, AD || BC, ∠DAB = 90°, AD = 10 cm and BC = 4 cm. If ABE is a quadrant of a circle, find the area of the shaded region. [Take π = 22/7]



Solution:

ar (trapezium) = 24.5 cm^2 … [Given

= 12(AD + BC) × AB = 24.5 …(i)

= (10 + 4) × AB = 24.5 × 2

= 14.(AB) = 49

AB = 4914 = 72 cm = r

$$\text{ar (quadrant ABE)} = \frac{90}{360}\pi r^2$$

$$= \frac{1}{4}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$

$$= \frac{77}{8} = 9.625 \text{ cm}^2 \qquad ...(ii)$$

∴ Area of the shaded region

= ar (trapezium) – ar (quadrant ABE)

= 24.5 – 9.625 = **14.875 cm^2**

SURFACE AREAS AND VOLUMES

Important formulae

- Surface Area of a cuboid of length (l), breadth (b), and height (h) = 2 ($lb + bh + lh$)
- Lateral Surface Area of cuboid = 2 $(l + b)h$
- Surface Area of a cube = 6 $\times l^2$ where l is the length
- Lateral Surface Area of cube = 4 $\times l^2$, where l is the length
- Volume of cube = l^3

- Curved Surface Area of a Cylinder = $2\pi rh$
- Total Surface Area of a Cylinder = $2\pi r\ (h + r)$
- Volume of Cylinder = $\pi r^2\ h$

- Lateral Surface Area of Cone = πrl
- Total surface area of cone = $\pi r\ (l + r)$
- Volume of Cone = ⅓ $(\pi r^2\ h)$
- Slant height , $l = \sqrt{r^2 + h^2}$

- Surface Area of Sphere = 4 πr^2
- Volume of Sphere = $\frac{4}{3}\ \pi r^3$

- Curved Surface Area of Hemisphere= $2\pi r^2$
- Total Surface Area of Hemisphere= 3 πr^2
- Volume of Hemisphere = $\frac{2}{3}\ \pi r^3$

One mark Questions

1. The radii of two cylinders are in the ratio 2:3 and their heights are in the ratio 5:3. The ratio of their volumes is

   (a) 27:20 (b) 20:27 (c) 9:4 (d) 4:9

2. If the volume of a cube is 1728 cm^3, the length of its edge is

   (a) 7cm (b) 12cm (c) 18cm (d) 19cm

3. The total surface area of a solid hemisphere of radius r is

   (a) $47\pi r^2$ (b) $2\pi r^2$ (c) $\frac{4}{3}\pi r^3$ (d) $3\pi r^2$

4. A conical tent with base radius 7m and height 24 m is made from 5m wide canvas. The length of the canvas used is ($\pi = \frac{22}{7}$)

   (a) 100m (b) 105m (c) 110m (d) 115m

5. Find the volume of a sphere of radius 21cm.

Two marks Questions

6. Find the CSA of a hemisphere whose radius is 7cm.
7. Find the radius of a cone of height 6cm and slant height is 10cm.
8. Find the surface area of the largest possible sphere carved out from a cylinder of radius 10mm and height 15mm.(use $\pi = 3.14$)
9. A cylinder has a base area 154cm^2. Find its volume if its height is 6cm.
10. Find the volume of a cuboid whose dimensions are 2.6cm, 8.2cm and 11cm.

Three marks Questions

11. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left.
12. A solid is in the shape of a cone standing on a hemisphere with both their radii being 1cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.
13. A cylinder and a cone have base radii 5cm and 3cm respectively and their respective heights are 4cm and 8cm. Find the ratio of their volumes.
14. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of the hemisphere?
15. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then calculate the rise of water level (in cm).

Five marks Questions

16. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14cm and the total height of the vessel is 13cm, find the inner surface area of the vessel.
17. A right circular cone of radius 4cm had a curved surface area of 47.1cm^2. Find the volume of the cone.
18. A solid iron pole consists of a cylinder of height 220cm and base diameter 24cm which is surmounted by another cylinder of height 60 cm and radius 8cm. Find the mass of the pole given that 1cm^3 of iron has approximately 8g mass.
19. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.
20. A tent is a combination of a cylinder and a cone. The slant height of the conical portion of the tent is10m and the height of the cylindrical portion is 7m. If the radius of the tent is 3.5m, find the area of the sheet used to make the tent.

Answers

1. (b) 20:27
2. (b) 12cm
3. (d) $3\pi r^2$
4. (c) 110m
5. 38808cm^3
6. CSA $= 2\pi r^2$

   $= 2\times \frac{22}{7} \times 7 \times 7$

   $=308$ cm^2
7. Radius$=\sqrt{10^2 - 6^2}$

   $= \sqrt{64}$

   $=8$cm
8. Radius of the sphere = radius of the cylinder= 10mm

   Surface area of the sphere $= 4\pi \times 10^2$ = 1256mm^2
9. Volume of the cylinder= $base\ area\ \times height$

   $= 154\times 6$ =924cm^3
10. Volume of the cuboid= $2.6\times 8.2 \times 11$

    = 234.52cm^3.
11. Volume of the wood left= volume of the cube – volume of the sphere

    $= 7^3 - \frac{4}{3}\pi 3.5^3$

    = 163.33cm^3
12. Volume$=\frac{1}{3}(\pi r^2 h) + \frac{2}{3}\pi r^3$

    $=\frac{1}{3}(\pi \times 1^2 \times 1) + \frac{2}{3}\pi 1^3$

    $= \frac{1}{3}\pi + \frac{2}{3}\pi$

    $= \pi$
13. Volume of cylinder: volume of cone $= \pi \times 5 \times 5 \times 4 : \pi \times 3 \times 3 \times 8$

    =25:18
14. Volume of hemisphere = Surface area of hemisphere …[Given

    $\frac{2}{3}\pi r^3 = 2\pi r^2 \Rightarrow \frac{1}{3}r = 1$

    $r = 3$

    ∴ Diameter of hemisphere = $2\times r = 2(3) = 6$ cm
15. Volume of Cylinder = Volume of Sphere

    $\pi R^2 h = \frac{4}{3}\pi r^3$

    $(18)^2 h = \frac{4}{3} \times (9)^3$...[∵ $R = \frac{36}{2} = 18$ cm; $r = \frac{18}{2} = 9$ cm

    ∴ $h = \frac{4}{3}\times\frac{9\times9\times9}{18\times18} = 3$ cm

16. Radius of the hemisphere=14/2 = 7cm
    Height of the cylinder =13-7 = 6cm
    Total inner surface area of vessel= inner surface area of hemisphere + inner surface area of cylinder = $2\pi r^2 + 2\pi rh$
    $=98\pi + 84\pi$
    $=182\pi$
    $= 572\ cm^2$

17. Let the height and the slant height of the cone be *h* cm and *l* cm respectively lt is given that the radius of the base is r =3 cm. It is also given that the curved surface area of the cone is 47.1 cm.
    $\pi rl = 47.1$

    $3.14 \times 3 \times l = 47.1$
    $l = \frac{47.1}{3.14\times 3} = 5cm$
    $l^2 = r^2 + h^2$
    $h = 4cm$
    Then using the formula $V = \frac{1}{3}\pi r^2 h$
    $=\frac{1}{3} \times 3.14 \times 3^2 \times 4$
    $=37.68cm^3$

18. Volume of the iron pole= sum of volume of the two cylinders
    $= \pi R^2 H + \pi r^2 h$
    $= \pi(12^2 \times 220 + 8^2 \times 60)$
    $= \pi \times 35520$
    Mass of the pole $= 35520\pi \times 8 = 892.26g$
19. Volume of the wood= volume of cuboid – 4×volume of cone
    $= 15\times 10 \times 3.5 - 4 \times \frac{1}{3}\pi 0.5^2 \times 1.4$
    $=523.53cm^3$

20. Area of the sheet = CSA of the cone+ CSA of the cylinder
    $= \pi rl + 2\pi rh$
    $= \pi r(l + 2h)$
    $=\frac{22}{7} \times 3.5 \times (10 + 14)$ $=11\times 24 = 264m^2$

STATISTICS

ARITHMETIC MEAN

Direct Method , $\bar{x} = \frac{\Sigma fixi}{\Sigma fi}$

Assumed Mean Method, $\bar{x} = a + \frac{\sum fidi}{\sum fi}$

MODE

Mode = $l + [\frac{(f1-f0)}{(2f1-f0-f2)}]h$

l = $lower\ limit\ of\ the\ modal\ class$

$f1$ = $frequency\ of\ modal\ class$

$f0$ = $frequency\ of\ the\ class\ preceding\ the\ modal\ class$

$f2$ = $frequency\ of\ the\ class\ following\ the\ modal$ class

h = $width(size)$ of the modal class

MEDIAN

Median = l+ [$\frac{\frac{N}{2}-cf}{f}$] h

l = $lower\ limit\ of\ the\ median\ class$

f = $frequency\ of\ median\ class$

h = $width(size)\ of\ the\ median\ class$

cf = $cumulative\ frequency\ of\ the\ class\ preceding\ the\ median\ class$

N = sum of all frequencies

THE EMPIRICAL RELATIONSHIP BETWEEN THE THREE MEASURES OF CENTRAL TENDENCY

$\mathbf{3\ median} = \mathbf{mode} + \mathbf{2}$ Mean

Section A(1 Mark Questions)

Q1.The median of a data is 20.If each item is increased by 2, the new median

(a) will remain same (b) increased by 2 (c) decreased by 2 (d) none of these

Q2. For the following distribution,the lower limit of the modal class is

Class Interval	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

(a) 15 (b) 25 (c) 30 (d) 35

Q3.The median of the following observations is 63.Find the value of Y if they are already arranged in

ascending order. 20 ,24 ,42, y, y+2, 73, 75, 80, 99

a) 2 (b) 61 (c) 63 (d) 65

Q4. Find the mode,when it is given that the mean and median are 10.5 and 9.6 respectevely.

(a) 7.8 (b) 19 (c) 10.5 (d) 9.5

Q5. If $d_i = x_i – 25$, $\Sigma f_i d_i = 200$ and $\Sigma f_i = 100$, then $\bar{x}$ is equal to

(a) 25 (b) 30 (c) 27 (d) 20

Section B (2Mark Questions)

Q1. The following data gives total household expenditure(in Rs) of labourers in a city. Find the modal expenditure.

Expenditure	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Workers	24	40	33	28	30	22	16	7

Q2. Convert the following distribution into a less than type distribution table.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

Q3. The A.M of the following distribution is 47. Determine the value of P.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	8	15	20	p	5

Q4. Find the mean of 32 numbers such that if the mean of 10 of them is 15 and the mean of 2 of them is 11.The last two numbers are 10.

Q5. For the following distribution, find the lower limit of the median class.

Marks	Below 10	Below 20	Below 30	Below40	Below 50	Below 60
No. of students	3	12	27	57	75	80

Section C(3Mark Questions)

Q1.If mode of the following distribution is 55, then find the value of x.

Class	0-15	15-30	30-45	45-60	60-75	75-90
frequency	10	7	x	15	10	12

Q2. Find the unknown entries a, b, c,d,e and f in the following distribution of heights in a class.

Height(in cm)	frequency	c.f
150-155	12	a
155-160	b	25
160-165	10	c
165-170	d	43
170-175	e	48
175-180	2	f
Total	50	

Q3. The following table shows the age distribution of cases of a certain disease admitted during a year in a particular hospital. Find the modal age.

Age(in years)	5-14	15-24	25-34	35-44	45-54	55-64	Total
No. of Cases	6	11	21	23	14	5	80

Q4.Find the mean of the following frequency distribution using assumed mean method.

Class	2-8	8-14	14-20	20-26	26-32
frequency	6	3	12	11	8

Q5. The following frequency distribution gives the monthly consumption(in units) of electricity by 68 consumers of a locality. Find the median consumption.

Monthly Consumption	65-85	85-105	105-125	125-145	145-165	165-185	185-205
No. of Consumers	4	5	13	20	14	8	4

Section D(5 Mark Questions)

Q1.If the median of the distribution is 28.5, find the values of x and y.

Class	0-10	10-20	20-30	30-40	40-50	50-60	Total
frequenc y	5	x	20	15	y	5	60

Q2. Find mean and mode of the given data. Also find the median using Empirical Formula.

Class	20-30	30-40	40-50	50-60	60-70
frequency	25	40	42	33	10

Q3. The mean of the following distribution is 53. Find the missing frequencies f1 and f2

Class	0-20	20-40	40-60	60-80	80-100	Total
frequency	15	f1	21	f2	17	100

Q4. Find the median of the following data:

Marks	Frequency
Less than 10	0
Less than 30	10
Less than 50	25
Less than 70	43
Less than 90	65
Less than 110	87
Less than 130	96
Less than 150	100

Q5. Compute the median from the following data.

Mid value	115	125	135	145	155	165	175	185	195
frequency	6	25	48	72	116	60	38	22	3

ANSWER KEY

1 MARK Question

(1) Increased by 2
(2) 15
(3) 61
(4) 7.8
(5) 27

2 MARK Question

(1) Mode = $l + [\frac{(f1-f0)}{(2f1-f0-f2)}]h$

$= 1500 + [\frac{(40-33)}{(2x40-33-24)}]500$

$=1500+ [\frac{7}{23}]500$

=1500+152.17 = 1652.17

Modal expenditure = ₹1652.17

(2)

Class	c.f
Less than 10	5
Less than 20	20
Less than 30	40
Less than 40	63
Less than 50	80
Less than 60	91
Less than 70	100

(3)

xi	fi	fixi
10	8	80
30	15	450
50	20	1000
70	P	70p
90	5	450
	Σfi=48+p	Σfixi=1980+70p

Mean = $\frac{\Sigma fixi}{\Sigma fi}$ ⇒ $\frac{1980+70p}{48+p}$ = 47

⇒70p -47p = 2256 -1980 = 276

⇒ 23p = 276 ⇒ p = 12

(4) The sum of 10 numbers =10X15 = 150

The sum of 20 numbers = 20X11 = 220

The sum of the last 2 numbers = 2X10 = 20

The sum of the 32 numbers = 150+220+20 = 390.

So the mean of the 32 numbers = 390/32 = 12.1875 or 12.19.

(5)

Classs	c.f
0-10	3
10-20	12
20-30	27
30-40	57
40-50	75
50-60	80

$\frac{N}{2}$ = 40

The class interval whose c.f is just greater than N/2 is 30-40.

So the median class is 30-40 and hence the lower limit of the median class is 30.

3 MARK Question

(1)

class	fi
0-15	10
15-30	7
30-45	X
45-60	15
60-75	10
75-90	12

Given that mode = 55. So modal class is 45-60, l = 45, f0= x, f1 =15, f2 =10, h=15

$$\text{Mode} = l + [\frac{(f1-f0)}{(2f1-f0-f2)}]h$$

$$55 = 45 + [\frac{(15-x)}{(2\,X\,15-x-10)}]\ 15$$

$$\frac{(15-x)}{(20-x)} = \frac{10}{15}$$

225- 15x = 200- 10x

⇒ 5x = 25 ⇒ x = 5

(2) a = 12

12 +b = 25 ⇒ b = 13
c = 25 + 10 = 35
c +d = 43 ⇒ 35 + d = 43 ⇒ d = 8
43 + e = 48 ⇒ e = 5
F = 48 +2 = 50

(3) modal class is 34.5-44.5 , l= 34.5 , f0=21, f1=23, f2=14, h= 10

$$\text{Mode} = l + [\frac{(f1-f0)}{(2f1-f0-f2)}]h = 34.5 + [\frac{(23-21)}{(2X23-21-14)}]10$$

$=34.5 + + \frac{(20\ )}{(11)}$ =34.5+ 1.8181….= 36.3181..

Modal age is 36. 32 years

(4)

class	f	xi	d=xi-a	fidi
2-8	6	5	-12	-72
8-14	3	11	-6	-18
14-20	12	17	0	0
20-26	11	23	6	66
26-32	8	29	12	96
	Σfi=40			Σfidi= 72

Assumed mean (a)= 17

Arithmetic mean ($\bar{x}$) = **a** + $\frac{(\Sigma fidi)}{(\Sigma fi)}$ = 17+$\frac{(72)}{(40)}$ = 17+ 1.8 = 18.8

(5)

Consumption(in units)	No. of consumers	c.f
65-85	4	4
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	8	64
185-205	4	68

$\frac{N}{2}$ = 34, median class= 125-145, l= 125, cf= 22, f= 20, h=20

Median = l+ [$\frac{\frac{N}{2}-cf}{f}$] h =125 + [$\frac{34-22}{20}$] 20 =125 +12 = 137

So Median consumption is 137 units.

5 MARK Question

(1)

Class	frequency	c.f
0-10	5	5
10-20	X	5+x
20-30	20	25+x
30-40	15	40+x
40-50	Y	40+x+y
50-60	5	45 +x +y
	45 + x+ y	

median= 28.5 , N =60, median class is 20-30, l=20, h=10, f=20, cf=5+x

$$\text{Median} = l + \left[\frac{\frac{N}{2} - cf}{f}\right] h$$

$$28.5 = 20 + \left[\frac{30-(5+x)}{20}\right]10$$

$25 - x = 17 \Rightarrow x = 8$

Also $x+y+45=60 \Rightarrow 8 + y + 45 = 60 \Rightarrow y = 7$

(2)

Class	frequency	Xi	fixi
20-30	25	25	625
30-40	40 – f0	35	1400
40-50	42 -f1	45	1890
50-60	33- f2	55	1815
60-70	10	65	650
Total	150		6380

$$\text{Mean} = \frac{6380}{150} = 42.53$$

$$\text{Mode} = \text{Mode} = l + \left[\frac{(f1-f0)}{(2f1-f0-f2)}\right]h = 40 + \left[\frac{42-40}{84-73}\right]10 = 41.81$$

3 median = 2 mean + mode

=(2 X 42.53) + 41.81 =126.87

Median = 42.29

(3)

class	frequency	xi	fixi
0-20	15	10	150
20-40	f1	30	30f1
40-60	21	50	1050
60-80	f2	70	70f2
80-100	17	90	1530
Total	100		2730 + 30f1 + 70f2

Sum of frequencies = 53+f1+f2= 100

f1+ f2 =47………….(1)

Given mean=53

$$\frac{2730+30f1+70f2}{100} = 53$$

2730 + 30f1 + 70f2 = 5300

30f1 + 70f2 = 2570…………….(2)

Solving (1) and (2) f1= 18, f2=29

(4)

Marks	Frequency	c.f
0-10	0	0
10- 30	10	10
30- 50	15	25
50-70	18	43
70- 90	22	65
90-110	22	87
110-130	9	96
130-150	4	100

$\frac{N}{2}$ = 50, median class:70-80, l= 70, cf = 43, f = 22, h =20

Median = l+ [$\frac{\frac{N}{2}-cf}{f}$] h

= 70+ [$\frac{50-43}{22}$] 20

=70+6.3636… = 76.36

(5)

The difference between the two consecutive values, h= 125- 115 = 10

Lower limit of a class = mid value - $\frac{h}{2}$

Upper limit of a class = mid value + $\frac{h}{2}$

Mid value	class	f	c.f
115	110-120	6	6
125	120-130	25	31
135	130-140	48	79
145	140-150	72	151
155	150-160	116	267
165	160-170	60	327
175	170-180	38	365
185	180-190	22	387
195	190-200	3	390

N= 390, $\frac{N}{2}$ = 195, median class : 150-160, l= 150, cf = 151, f= 116, h=10

Median = l+ [$\frac{\frac{N}{2}-cf}{f}$] h = 150 +[$\frac{195-151}{116}$]10

= 150 + $\frac{440}{116}$

= 150 + 3.79 = 153.79

PROBABILITY

$$\Rightarrow \quad P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$\Rightarrow$ $P(E) + P(\underline{E}) = 1$

NOTE :

- *P(E)* means *Probability of getting an event*

  (It is read as P of E)

- *P* ($\underline{E}$) means *Probability of not getting an event.*

  (It is read as P of E complement)

SECTION – A (1 Mark each)

1. A letter of the English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant?
2. Find the probability of getting an even number when a die is thrown once?
3. In tossing a die, what is the probability of getting a number less than 4?
4. Can 1.67 be the probability of an event?
5. A number is selected at random from 1 to 20. Find the probability that it is a prime number?

SECTION – B (2 Marks each)

6. A bag contains 3 red, 4 green and 5 white candles, one candle is drawn at random from the bag, find the probability that the candle is not red?

7. Two dice are rolled simultaneously. Find the probability that the sum of numbers appearing on top is 10?

8. Hari tossed two different coins simultaneously. What is the probability that

he gets:

(i) atleast one head (ii) one head and one tail

9. A letter of English alphabet is chosen at random, find the probability that the letter so chosen is :

(i) a vowel (ii) a consonant

10. A bag contains 4 red balls, 5 black balls and 6 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is :

(i) White (ii) Red

(iii) Red or Black (iv) Not Black

SECTION – C (3 Marks each)

11. A box contains 19 balls bearing numbers 1, 2,….18, 19. A ball is drawn at random from the box. Find the probability that the number on the ball is:

(i) a composite number (ii) an even number

(iii) divisible by 3 or 5

12. Two dice are thrown at the same time. Find the probability of getting:

(i) same number on both dice

(ii) even number on both dice

13. Three coins are tossed simultaneously once. Find the probability of getting:

(i) Atleast one tail (ii) No tail

14. One card is drawn from a well- shuffled deck of 52 cards. Find the probability of getting:

(i) A spade (ii) A red face card (iii) Either a king or black cards

15. Out of 200 bulbs in a box, 15 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective?

SECTION – D (5 Marks each)

16. Two friends Ejoe and Ron have some savings in their piggy bank. They decided to count the total coins they both had. After counting they find that they have fifty ₹1 coins, forty eight ₹ 2 coins, thirty six ₹ 5 coins, twenty eight ₹10 coins and eight ₹ 20 coins. Now, they said to Sandra, their another friend, to choose a coin randomly.

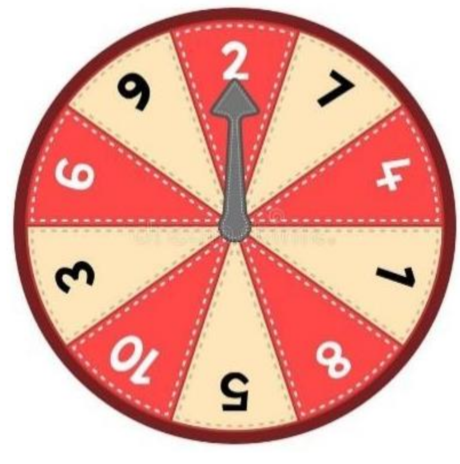
(i) Find the probability of getting a denomination of ₹10.

(ii) Find the probability of getting a denomination of ₹ 2 or ₹ 5.

(iii) Find the probability of getting a denomination of ₹1.

(iv) Find the probability of getting a denomination of ₹ 20.

17. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. (see figure) and there are equally likely outcomes.



What is the probability that it will point at :

(i) the number 8.

(ii) an odd number.

(iii) a number greater than 2.

(iv) number which is a multiple of 3.

(v) Which mathematical concept is used in the above problem.

18. All the black face cards are removed from a pack of 52 cards. Find the probability of getting :

(i) Face card (ii) Red Card (iii) King

(iv) Black Card (v) Queen

19. Three coins are tossed simultaneously. Find the probability of getting:

(i) exactly 2 heads

(ii) at least 1 head

(iii) at most 2 tails

(iv) exactly 3 tails

(v) at least 2 heads

20. Sereena goes to market for buying an aquarium for her house. She asked the shopkeeper to put some fish in the aquarium. The shopkeeper takes out 13 guppy fish, 18 flowerhorn fish, 12 koi fish and 11 angel fish from the big tank he had & put them in the aquarium that Sereena had bought. Now, Sereena selects a fish at random.

On the basis of above information, answer the following questions.

(i) If total number of male fish in the aquarium is 36, then what is the probability of selecting a female fish?

(ii) What is the probability of selecting a guppy fish ?

(iii) What is the probability of not selecting an angel fish ?

(iv) What is the probability of selecting a flowerhorn fish ?

ANSWERS

SECTION – A

1. Total outcomes = 26

Consonant = b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z = 21

∴ P (getting a consonant) = $\frac{21}{26}$

2. Total outcomes = 6

Even numbers = 2, 4, 6

∴ P (getting an even number) = $\frac{3}{6} = \frac{1}{2}$

3. Total outcomes = 6

Numbers less than 4 = 1, 2, 3

∴ P (getting a number less than 4) = $\frac{3}{6} = \frac{1}{2}$

4. No. Since the probability of an event cannot be more than 1.

5. Number of possible outcomes = 20

Prime numbers are = 2, 3, 5, 7, 11, 13, 17, 19 = 8

∴ P (getting a prime number) = $\frac{8}{20} = \frac{2}{5}$

SECTION – B

6. Total number of candles = Red candles + Green candles + White candles

= 3 + 4 + 5 = 12

Number of red candles = 3

Number of green candles = 4

Number of white candles = 5

No: of candles which is not red = No: of green candles + No: of white candles

$= 4 + 5 = 9$

∴ P (getting a candle which is not red) = $\frac{9}{12} = \frac{3}{4}$

OR

Total number of candles = 3 + 4 + 5 = 12

Number of red candles = 3

P (getting a candle which is red) = $\frac{3}{12} = \frac{1}{4}$

[P (E) + P ($\underline{E}$) = 1 ⇒ $P\left(\underline{E}\right) = 1 - P\ (E)$]

∴ P (getting a candle which is not red) = 1 – P (getting a candle which is red)

$= 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$

7. When two dice are thrown total number of outcomes = 36

If sum of both faces should be 10 they are { (4, 6) , (6, 4) , (5, 5) } = 3

∴ P (E) $= \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes} = \frac{3}{36} = \frac{1}{12}$

8. Total number of outcomes = { HH, TT, HT, TH }

(i) P (getting atleast one head) = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes} = \frac{3}{4}$

(ii) P (getting one head and one tail) = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

$= \frac{2}{4} = \frac{1}{2}$

9. Total number of outcomes = 26

Vowels = a, e, i, o, u = 5

Consonant = b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z = 21

(i) a vowel

∴ P (getting a vowel) = $\frac{5}{26}$

(ii) a consonant

∴ P (getting a consonant) = $\frac{21}{26}$

10. Total number of balls = 4 + 5 + 6 = 15

Number of red balls = 4

Number of black balls = 5

Number of white balls = 6

(i) ∴ P (getting a white ball) $= \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes} = \frac{6}{15} = \frac{2}{5}$

(ii) ∴ P (getting a red ball) $= \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes} = \frac{4}{15}$

(iii) ∴ P (getting a red or black ball) = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

$= \frac{9}{15} = \frac{3}{5}$

(iv) ∴ P (not getting a black ball) = 1 – P (getting a black ball)

$= 1 - \frac{5}{15} = \frac{15-5}{15}$

$= \frac{10}{15} = \frac{2}{3}$

<u>SECTION – C</u>

11. Total number of balls = 19

(Bearing numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19)

P (E) = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

(i) Total number of composite numbers = 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 = 10

∴ P (getting a composite number) = $\frac{10}{19}$

(ii) Total number of even numbers = 2, 4, 6, 8, 10, 12, 14, 16, 18 = 9

∴ P (getting an even number) = $\frac{9}{19}$

(iii) Numbers divisible by 3 or 5 = 3, 5, 6, 9, 10, 12, 15, 18 = 8

∴ P (getting a number divisible by 3 or 5) = $\frac{8}{19}$

12. When two dice are thrown total number of outcomes = 36

P (E) $= \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

(i) Possible outcomes = { (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) }

∴ P (getting same number on both dice) = $\frac{6}{36} = \frac{1}{6}$

(ii) Possible outcomes = {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)}

∴ P (getting even number on both dice) = $\frac{9}{36} = \frac{1}{4}$

13. Possible outcomes = { HHH, TTT, HTT, THT, TTH, THH, HHT, HTH }

$$P(E) = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

(i) P (getting atleast one tail) = $\frac{7}{8}$

(ii) P (getting no tail) = $\frac{1}{8}$

14. Total number of cards = 52

$$P(E) = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

(i) Number of spade cards = 13

∴ P (getting a spade) = $\frac{13}{52} = \frac{1}{4}$

(ii) Number of red face card cards = 3 + 3 = 6

[*Face cards – King, Queen and Jack ; Red cards = Diamond and Heart*]

∴ P (getting a red face card) = $\frac{6}{52} = \frac{3}{26}$

(iii) Either a king or black cards = 2 + 13 + 13 = 28

[*Black cards = 13 + 13 = 26 ; and 2 kings from red card set*]

∴ P (getting either a king or black cards) = $\frac{28}{52} = \frac{7}{13}$

15. Total number of bulbs in the box = 200

Number of defective bulbs = 15

Number of good bulbs = 200 – 15 = 185

P (getting a bulb which is not defective) = $\frac{185}{200} = \frac{37}{40}$

[*Bulb which is not defective means good bulb*]

SECTION – D

16. Total number of ₹ 1 coin = 50

Total number of ₹ 2 coin = 48

Total number of ₹ 5 coin = 36

Total number of ₹10 coin = 28

Total number of ₹ 20 coin = 8

∴ Total number of coins = 50 + 48 + 36 + 28 + 8 = 170

(i) P (getting a denomination of ₹10) = $\frac{28}{170} = \frac{14}{85}$

(ii) P (getting a denomination of ₹ 2 or ₹ 5) = $\frac{84}{170} = \frac{42}{85}$

[*Total = 48 + 36 = 84*]

(iii) P (getting a denomination of ₹1) = $\frac{50}{170} = \frac{5}{17}$

(iv) P (getting a denomination of ₹ 20) = $\frac{8}{170} = \frac{4}{85}$

17. Total number of points = 8 {*1, 2, 3, 4, 5, 6, 7, 8, 9, 10*}

P (E) = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

(i) P (getting the number 8) = $\frac{1}{10}$

(ii) P (getting an odd number) = $\frac{5}{10} = \frac{1}{2}$

[*odd numbers* - *1, 3, 5, 7, 9. So total odd numbers = 5*]

(iii) P (getting a number greater than 2) = $\frac{8}{10} = \frac{4}{5}$

[*numbers greater than 2 are 3, 4, 5, 6, 7, 8, 9, 10.*

So, total numbers greater than 2 = 8

(iv) P (getting a multiple of 3) = $\frac{3}{10}$

[*multiple of 3 are 3, 6, 9. So total numbers that are multiple of 3 = 3*]

(v) Probability

18. Since all the black face cards are removed,

Total cards remaining = 52 – (3 +3)

= 52 – 6 = 46

(i) P (getting a face card) = $\frac{6}{46} = \frac{3}{23}$

[*Diamond face card – 3 ; Heart face card – 3 ; ∴ Total face card = 3 + 3 = 6*]

(ii) P (getting a red card) = $\frac{26}{46} = \frac{13}{23}$

[*Number of red cards = 13 diamond cards + 13 heart cards = 13 + 13 = 26*]

(iii) P (getting king card) = $\frac{2}{46} = \frac{1}{23}$

[*Black face cards are removed. 2 red king (Diamond and Heart)*]

(iv) P (getting a black card) = $\frac{20}{46} = \frac{10}{23}$

[*Black face cards are removed. Balance black cards = 26 - 6 = 20*]

(v) P (getting queen) = $\frac{2}{46} = \frac{1}{23}$

[*Black face cards are removed. 2 red queen (Diamond and Heart)*]

19. Possible outcomes = { HHH, TTT, HTT, THT, TTH, THH, HHT, HTH }

P (E) = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

(i) P (getting exactly 2 heads) = $\frac{3}{8}$

(ii) P (getting at least 1 head) = $\frac{7}{8}$

(iii) P (getting at most 2 tails) = $\frac{7}{8}$

(iv) P (getting exactly 3 tails) = $\frac{1}{8}$

(v) P (getting at least 2 heads) = $\frac{4}{8}$ = $\frac{1}{2}$

20. Total number of fish in the aquarium = 13 + 18 +12 + 11 = 54

Number of guppy fish = 13

Number of flowerhorn fish = 18

Number of koi fish = 12

Number of angel fish = 11

(i) Number of male fish in the aquarium = 36

Number of female fish in the aquarium = 54 – 36 = 18

∴ P (of selecting a female fish) = $\frac{18}{54}$ = $\frac{1}{3}$

(ii) P (of selecting a guppy fish) = $\frac{13}{54}$

(iii) Total number of fish other than angel fish = 13 + 18 +12 = 43

[*Not selecting an angel fish means slecting a fish other than angel fish*]

∴ P (of not selecting an angel fish) = $\frac{43}{54}$

(iv) P (of selecting a flowerhorn fish is) = $\frac{18}{54}$ = $\frac{1}{3}$

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